# Lesson 6: Dividing by $x - a$ and by $x + a$

## Classwork

### Opening Exercise

Find the following quotients, and write the quotient in standard form.

a. $\frac{x^2-9}{x-3}$

b. $\frac{x^3-27}{x-3}$

c. $\frac{x^4-81}{x-3}$

## Exercise 1

1. Use patterns to predict each quotient. Explain how you arrived at your prediction, and then test it by applying the reverse tabular method or long division.

   a. $\frac{x^2-144}{x-12}$

   b. $\frac{x^3-8}{x-2}$

   c. $\frac{x^3-125}{x-5}$

   d. $\frac{x^6-1}{x-1}$

## Example 1

What is the quotient of $\frac{x^2-a^2}{x-a}$? Use the reverse tabular method or long division.

## Exercises 2–4

2. Work with your group to find the following quotients.

   a. $\frac{x^3-a^3}{x-a}$

   b. $\frac{x^4-a^4}{x-a}$

3. Predict without performing division whether or not the divisor will divide into the dividend without a remainder for the following problems. If so, find the quotient. Then check your answer.

   a. $\frac{x^2-a^2}{x+a}$

   b. $\frac{x^3-a^3}{x+a}$

   c. $\frac{x^2+a^2}{x+a}$

   d. $\frac{x^3+a^3}{x+a}$

4.

a. Find the quotient $\frac{x^n - 1}{x - 1}$ for $n = 2, 3, 4,$ and $8$

b. What patterns do you notice?

c. Use your work in part (a) to write an expression equivalent to $\frac{x^n - 1}{x - 1}$ for any integer $n > 1$.

**Lesson Summary**

Based on the work in this lesson, it can be concluded that the following statements are true for all real values of $x$ and $a$:

$$x^2 - a^2 = (x - a)(x + a)$$
$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$
$$x^3 + a^3 = (x + a)(x^2 - ax + a^2),$$

and it seems that the following statement is also an identity for all real values of $x$ and $a$:

$x^n - 1 = (x - 1)(x^{n-1} + x^{n-2} + x^{n-3} + \cdots + x^1 + 1)$, for integers $n > 1$.

## Problem Set

1. Compute each quotient.

   a. $\frac{x^2-625}{x-25}$

   b. $\frac{x^3+1}{x+1}$

   c. $\frac{x^3-\frac{1}{8}}{x-\frac{1}{2}}$

   d. $\frac{x^2-0.01}{x-0.1}$

2. In the next exercises, you can use the same identities you applied in the previous problem. Fill in the blanks in the problems below to help you get started. Check your work by using the reverse tabular method or long division to make sure you are applying the identities correctly.

   a. $\frac{16x^2-121}{4x-11} = \frac{(\_\_)^2-(\_\_)^2}{4x-11} = (\_\_) + 11$

   b. $\frac{25x^2-49}{5x+7} = \frac{(\_\_)^2-(\_\_)^2}{5x+7} = (\_\_) - (\_\_) =$ ____________

   c. $\frac{8x^3-27}{2x-3} = \frac{(\_\_)^3-(\_\_)^3}{2x-3} = (\_\_)^2 + (\_\_)(\_\_) + (\_\_)^2 =$ ____________________

3. Show how the patterns and relationships learned in this lesson could be applied to solve the following arithmetic problems by filling in the blanks.

   a. $\frac{625-81}{16} = \frac{(\_\_)^2-(9)^2}{25-(\_\_)} = (\_\_) + (\_\_) = 34$

   b. $\frac{1000-27}{7} = \frac{(\_\_)^3-(\_\_)^3}{(\_\_)-3} = (\_\_)^2 + (10)(\_\_) + (\_\_)^2 = \_\_\_\_\_\_\_$

   c. $\frac{100-9}{7} = \frac{(\_\_)^2-(\_\_)^2}{(\_\_)-3} = \_\_\_\_\_\_\_$

   d. $\frac{1000+64}{14} = \frac{(\_\_)^3+(\_\_)^3}{(\_\_)+(\_\_)} = (\_\_)^2 - (\_\_)(\_\_) + (\_\_)^2 = \_\_\_\_\_\_\_$

4. Apply the identities from this lesson to compute each quotient. Check your work using the reverse tabular method or long division.

   a. $\frac{16x^2-9}{4x+3}$

   b. $\frac{81x^2-25}{18x-10}$

   c. $\frac{27x^3-8}{3x-2}$

5. Extend the patterns and relationships you learned in this lesson to compute the following quotients. Explain your reasoning, and then check your answer by using long division or the tabular method.

   a. $\frac{8+x^3}{2+x}$

   b. $\frac{x^4-y^4}{x-y}$

   c. $\frac{27x^3+8y^3}{3x+2y}$

   d. $\frac{x^7-y^7}{x-y}$