

## Lesson 31: Systems of Equations

## Classwork

## Exploratory Challenge 1

- a. Sketch the lines given by  $x + y = 6$  and  $-3x + y = 2$  on the same set of axes to solve the system graphically. Then solve the system of equations algebraically to verify your graphical solution.

- b. Suppose the second line is replaced by the line with equation  $x + y = 2$ . Plot the two lines on the same set of axes, and solve the pair of equations algebraically to verify your graphical solution.

- c. Suppose the second line is replaced by the line with equation  $2x = 12 - 2y$ . Plot the lines on the same set of axes, and solve the pair of equations algebraically to verify your graphical solution.
- d. We have seen that a pair of lines can intersect in 1, 0, or an infinite number of points. Are there any other possibilities?

**Exploratory Challenge 2**

- a. Suppose that instead of equations for a pair of lines, you were given an equation for a circle and an equation for a line. What possibilities are there for the two figures to intersect? Sketch a graph for each possibility.
- b. Graph the parabola with equation  $y = x^2$ . What possibilities are there for a line to intersect the parabola? Sketch each possibility.

- c. Sketch the circle given by  $x^2 + y^2 = 1$  and the line given by  $y = 2x + 2$  on the same set of axes. One solution to the pair of equations is easily identifiable from the sketch. What is it?
- d. Substitute  $y = 2x + 2$  into the equation  $x^2 + y^2 = 1$ , and solve the resulting equation for  $x$ .
- e. What does your answer to part (d) tell you about the intersections of the circle and the line from part (c)?

### Exercises

1. Draw a graph of the circle with equation  $x^2 + y^2 = 9$ .
- a. What are the solutions to the system of circle and line when the circle is given by  $x^2 + y^2 = 9$ , and the line is given by  $y = 2$ ?

- b. What happens when the line is given by  $y = 3$ ?
- c. What happens when the line is given by  $y = 4$ ?
2. By solving the equations as a system, find the points common to the line with equation  $x - y = 6$  and the circle with equation  $x^2 + y^2 = 26$ . Graph the line and the circle to show those points.

3. Graph the line given by  $5x + 6y = 12$  and the circle given by  $x^2 + y^2 = 1$ . Find all solutions to the system of equations.
4. Graph the line given by  $3x + 4y = 25$  and the circle given by  $x^2 + y^2 = 25$ . Find all solutions to the system of equations. Verify your result both algebraically and graphically.
5. Graph the line given by  $2x + y = 1$  and the circle given by  $x^2 + y^2 = 10$ . Find all solutions to the system of equations. Verify your result both algebraically and graphically.

6. Graph the line given by  $x + y = -2$  and the quadratic curve given by  $y = x^2 - 4$ . Find all solutions to the system of equations. Verify your result both algebraically and graphically.

**Lesson Summary**

Here are some steps to consider when solving systems of equations that represent a line and a quadratic curve.

1. Solve the linear equation for  $y$  in terms of  $x$ . This is equivalent to rewriting the equation in slope-intercept form. Note that working with the quadratic equation first would likely be more difficult and might cause the loss of a solution.
2. Replace  $y$  in the quadratic equation with the expression involving  $x$  from the slope-intercept form of the linear equation. That will yield an equation in one variable.
3. Solve the quadratic equation for  $x$ .
4. Substitute  $x$  into the linear equation to find the corresponding value of  $y$ .
5. Sketch a graph of the system to check your solution.

**Problem Set**

1. Where do the lines given by  $y = x + b$  and  $y = 2x + 1$  intersect?
2. Find all solutions to the following system of equations.

$$\begin{aligned}(x - 2)^2 + (y + 3)^2 &= 4 \\ x - y &= 3\end{aligned}$$

Illustrate with a graph.

3. Find all solutions to the following system of equations.

$$\begin{aligned}x + 2y &= 0 \\ x^2 - 2x + y^2 - 2y - 3 &= 0\end{aligned}$$

Illustrate with a graph.

4. Find all solutions to the following system of equations.

$$\begin{aligned}x + y &= 4 \\ (x + 3)^2 + (y - 2)^2 &= 10\end{aligned}$$

Illustrate with a graph.

5. Find all solutions to the following system of equations.

$$\begin{aligned}y &= -2x + 3 \\ y &= x^2 - 6x + 3\end{aligned}$$

Illustrate with a graph.

6. Find all solutions to the following system of equations.

$$\begin{aligned}-y^2 + 6y + x - 9 &= 0 \\ 6y &= x + 27\end{aligned}$$

Illustrate with a graph.

7. Find all values of  $k$  so that the following system has two solutions.

$$\begin{aligned}x^2 + y^2 &= 25 \\ y &= k\end{aligned}$$

Illustrate with a graph.

8. Find all values of  $k$  so that the following system has exactly one solution.

$$\begin{aligned}y &= 5 - (x - 3)^2 \\ y &= k\end{aligned}$$

Illustrate with a graph.

9. Find all values of  $k$  so that the following system has no solutions.

$$\begin{aligned}x^2 + (y - k)^2 &= 36 \\ y &= 5x + k\end{aligned}$$

Illustrate with a graph.