Lesson 32: Graphing Systems of Equations

Classwork

Opening Exercise

Given the line $y = 2x$, is there a point on the line at a distance 3 from $(1, 3)$? Explain how you know.

Draw a graph showing where the point is.

Exercise 1

Solve the system $(x - 1)^2 + (y - 2)^2 = 2^2$ and $y = 2x + 2$.

What are the coordinates of the center of the circle?

What can you say about the distance from the intersection points to the center of the circle?

Using your graphing tool, graph the line and the circle.

Example 1

Rewrite $x^2 + y^2 - 4x + 2y = -1$ by completing the square in both x and y. Describe the circle represented by this equation.

Using your graphing tool, graph the circle.

In contrast, consider the following equation: $x^2 + y^2 - 2x - 8y = -19$

What happens when you use your graphing tool with this equation?

Exercise 2

Consider a circle with radius 5 and another circle with radius 3. Let d represent the distance between the two centers. We want to know how many intersections there are of these two circles for different values of d. Draw figures for each case.

a. What happens if $d = 8$?

b. What happens if $d = 10$?

c. What happens if $d = 1$?

d. What happens if $d = 2$?

e. For which values of d do the circles intersect in exactly one point? Generalize this result to circles of any radius.

f. For which values of d do the circles intersect in two points? Generalize this result to circles of any radius.

g. For which values of d do the circles not intersect? Generalize this result to circles of any radius.

Example 2

Find the distance between the centers of the two circles with equations below, and use that distance to determine in how many points these circles intersect.

$$x^2 + y^2 = 5$$
$$(x-2)^2 + (y-1)^2 = 3$$

Exercise 3

Use the distance formula to show algebraically and graphically that the following two circles do not intersect.

$$(x-1)^2 + (y+2)^2 = 1$$
$$(x+5)^2 + (y-4)^2 = 4$$

Example 3

Point $A(3, 2)$ is on a circle whose center is $C(-2, 3)$. What is the radius of the circle?

What is the equation of the circle? Graph it.

Use the fact that the tangent at $A(3, 2)$ is perpendicular to the radius at that point to find the equation of the tangent line. Then graph it.

Find the coordinates of point B, the second intersection of the $\overleftrightarrow{AC}$ and the circle.

What is the equation of the tangent to the circle at $(-7, 4)$? Graph it as a check.

The lines $y = 5x + b$ are parallel to the tangent lines to the circle at points A and B. How is the y-intercept b for these lines related to the number of times each line intersects the circle?

Problem Set

1. Use the distance formula to find the distance between the points $(-1, -13)$ and $(3, -9)$.

2. Use the distance formula to find the length of the longer side of the rectangle whose vertices are $(1, 1)$, $(3, 1)$, $(3, 7)$, and $(1, 7)$.

3. Use the distance formula to find the length of the diagonal of the square whose vertices are $(0, 0)$, $(0, 5)$, $(5, 5)$, and $(5, 0)$.

Write an equation for the circles in Exercises 4–6 in the form $(x-h)^2 + (y-k)^2 = r^2$, where the center is (h, k) and the radius is r units. Then write the equation in the standard form $x^2 + ax + y^2 + by + c = 0$, and construct the graph of the equation.

4. A circle with center $(4, -1)$ and radius 6 units.

5. A circle with center $(-3, 5)$ tangent to the x-axis.

6. A circle in the third quadrant, radius 1 unit, tangent to both axes.

7. By finding the radius of each circle and the distance between their centers, show that the circles $x^2 + y^2 = 4$ and $x^2 - 4x + y^2 - 4y + 4 = 0$ intersect. Illustrate graphically.

8. Find the points of intersection of the circles $x^2 + y^2 - 15 = 0$ and $x^2 - 4x + y^2 + 2y - 5 = 0$. Check by graphing the equations.

9. Solve the system $y = x^2 - 2$ and $x^2 + y^2 = 4$. Illustrate graphically.

10. Solve the system $y = 2x - 13$ and $y = x^2 - 6x + 3$. Illustrate graphically.