Topic D

A Surprise from Geometry—Complex Numbers Overcome All Obstacles

N-CN.A.1, N-CN.A.2, N-CN.C.7, A-REI.A.2, A-REI.B.4b, A-REI.C.7

Focus Standards:		
	N-CN.A.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
	N-CN.A.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
	N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
	A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
	A-REI.B.4	Solve quadratic equations in one variable.
		Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b.
	A-REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.
Instructional Days:	5	
Lesson 36:	Overcoming a Third Obstacle to Factoring—What If There Are No Real Number Solutions? (P)1	
Lesson 37:	A Surprising Boost from Geometry (P)	
Lesson 38:	Complex Numbers as Solutions to Equations (P)	
Lesson 39:	Factoring Extended to the Complex Realm (P)	
Lesson 40:	Obstacles Resolved—A Surprising Result (S)	

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[1]Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

In In Topic D, students extend their facility with finding zeros of polynomials to include complex zeros. Lesson 36 presents a third obstacle to using factors of polynomials to solve polynomial equations. Students begin by solving systems of linear and nonlinear equations to which no real solutions exist and then relate this to the possibility of quadratic equations with no real solutions. Lesson 37 introduces complex numbers through their relationship to geometric transformations. That is, students observe that scaling all numbers on a number line by a factor of -1 turns the number line out of its one-dimensionality and rotates it $180°$ through the plane. They then answer the question, "What scale actor could be used to create a rotation of $90°$?" In Lesson 38, students discover that complex numbers have real uses; in fact, they can be used in finding real solutions of polynomial equations. In Lesson 39, students develop facility with properties and operations of complex numbers and then apply that facility to factor polynomials with complex zeros. Lesson 40 brings the module to a close with the result that every polynomial can be rewritten as the product of linear factors, which is not possible without complex numbers. Even though standards **N-CN.C.8** and **N-CN.C.9** are not assessed at the Algebra II level, they are included instructionally to develop further conceptual understanding.