Lesson 11: Transforming the Graph of the Sine Function

Classwork

Opening Exercise

Explore your assigned parameter in the sinusoidal function $f(x) = A\sin\big(\omega(x-h)\big)+k$. Select several different values for your assigned parameter, and explore the effects of changing the parameter's value on the graph of the function compared to the graph of $f(x)=\sin(x)$. Record your observations in the table below. Include written descriptions and sketches of graphs.

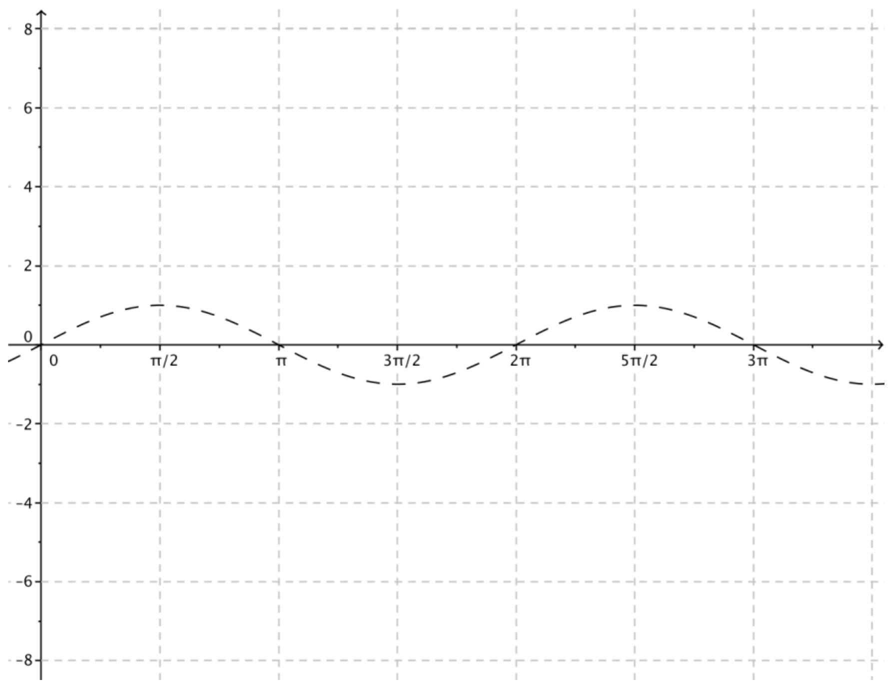
***A*-Team**	**ω-Team**
$f(x) = A\sin(x)$	$f(x)=\sin(\omega x)$
Suggested A values: $2, 3, 10, 0, -1, -2, \frac{1}{2}, \frac{1}{5}, -\frac{1}{3}$	Suggested ω values: $2, 3, 5, \frac{1}{2}, \frac{1}{4}, 0, -1, -2, \pi, 2\pi, 3\pi, \frac{\pi}{2}, \frac{\pi}{4}$

***k*-Team**	***h*-Team**
$f(x) = \sin(x) + k$ Suggested k values: $2, 3, 10, 0, -1, -2, \frac{1}{2}, \frac{1}{5}, -\frac{1}{3}$	$f(x) = \sin(x - h)$ Suggested h values: $\pi, -\pi, \frac{\pi}{2}, -\frac{\pi}{4}, 2\pi, 2, 0, -1, -2, 5, -5$

Example

Graph the following function:

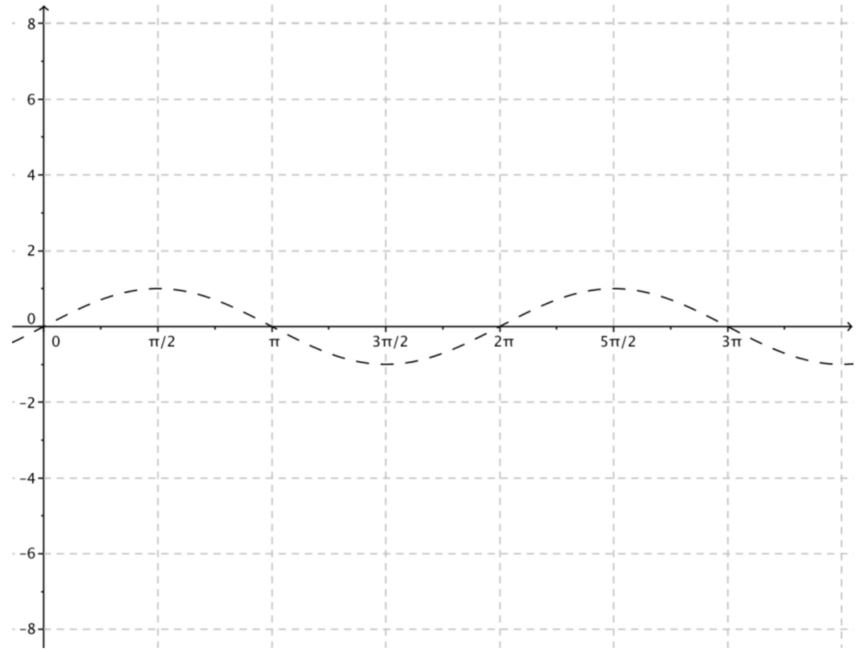
$$f(x) = 3\sin\left(4\left(x - \frac{\pi}{6}\right)\right) + 2.$$



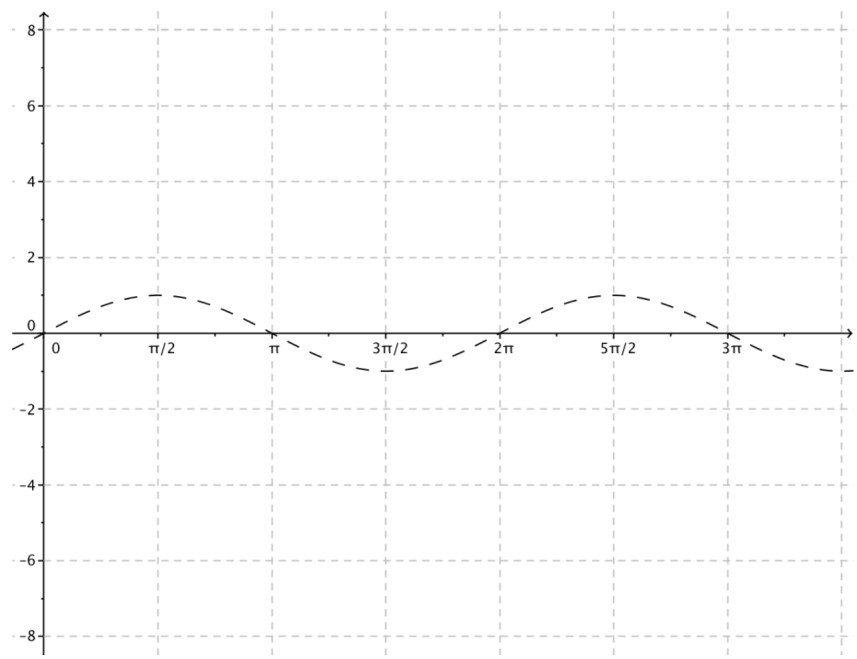
Exercise

For each function, indicate the amplitude, frequency, period, phase shift, vertical translation, and equation of the midline. Graph the function together with a graph of the sine function $f(x) = \sin(x)$ on the same axes. Graph at least one full period of each function.

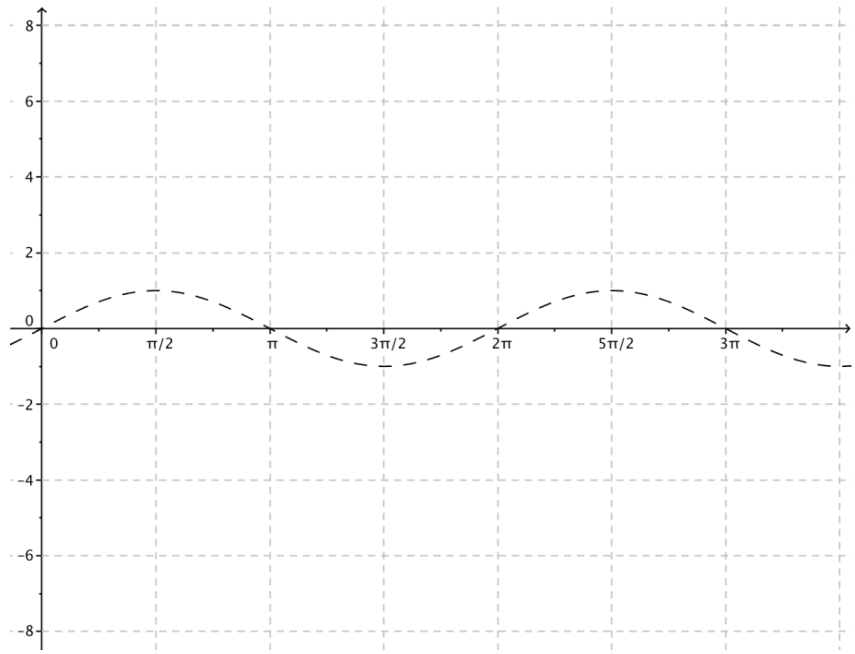
a. $g(x) = 3\sin(2x) - 1$



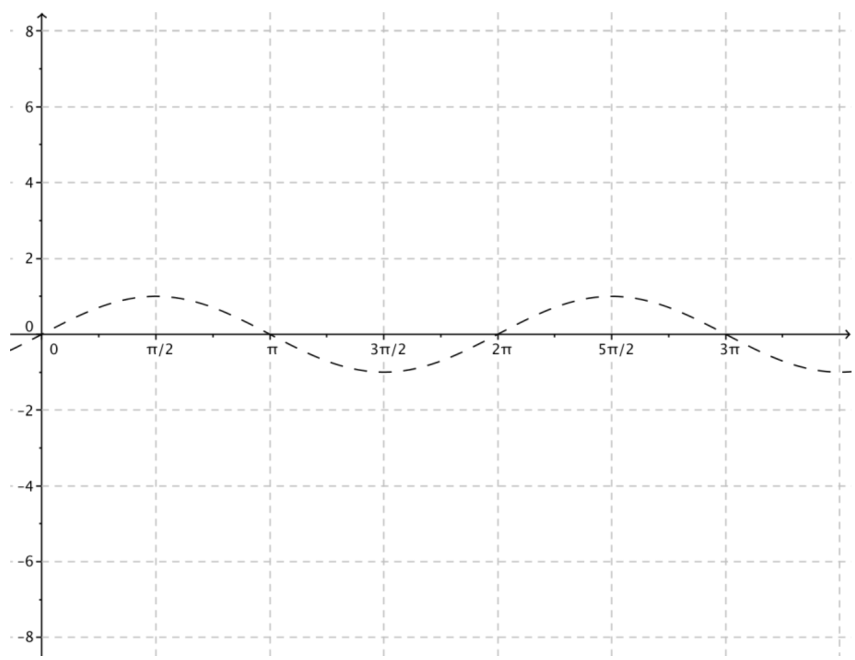
b. $g(x) = \frac{1}{2}\sin\left(\frac{1}{4}(x + \pi)\right)$



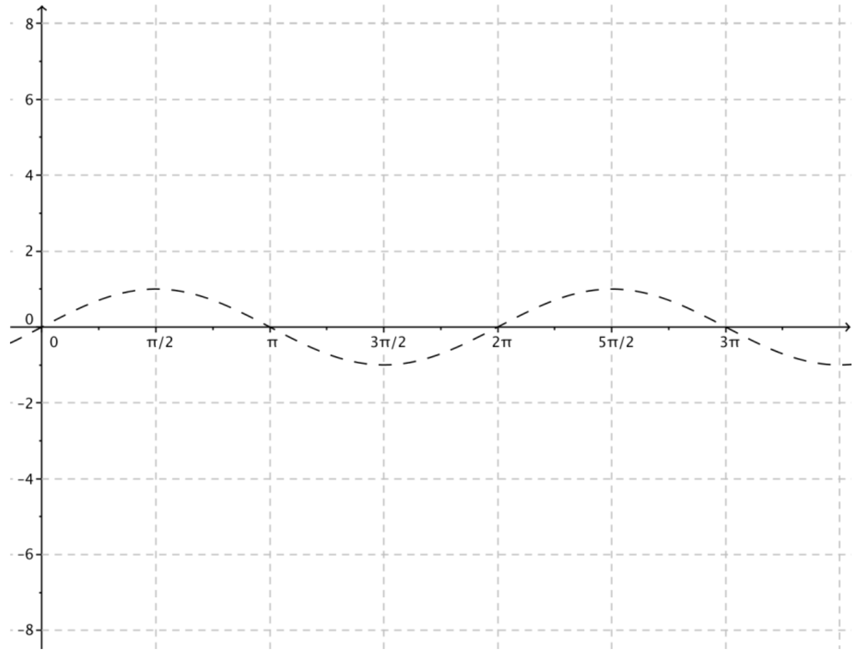
c. $g(x) = 5\sin(-2x) + 2$



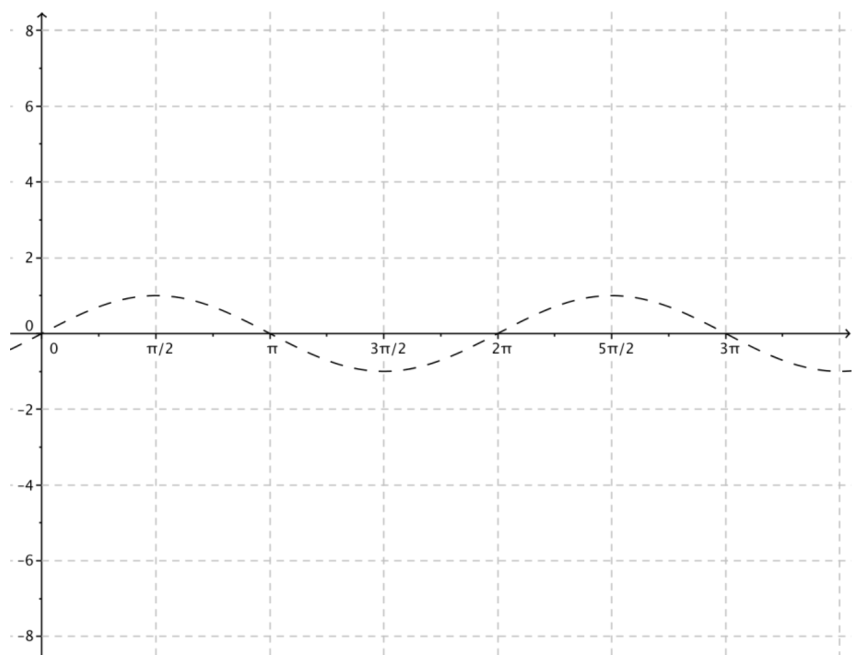
d. $g(x) = -2\sin\left(3\left(x + \frac{\pi}{6}\right)\right)$



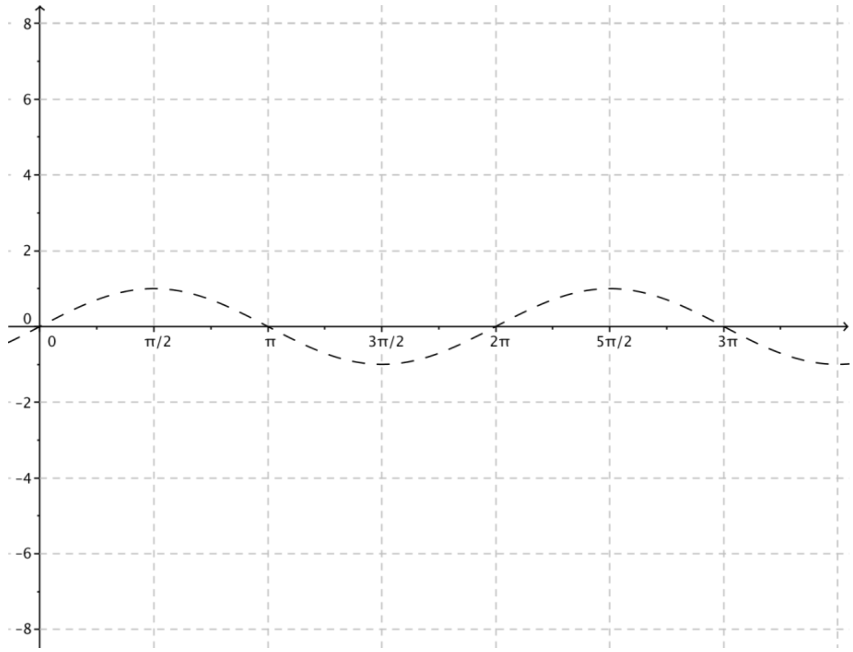
e. $g(x) = 3\sin(x + \pi) + 3$



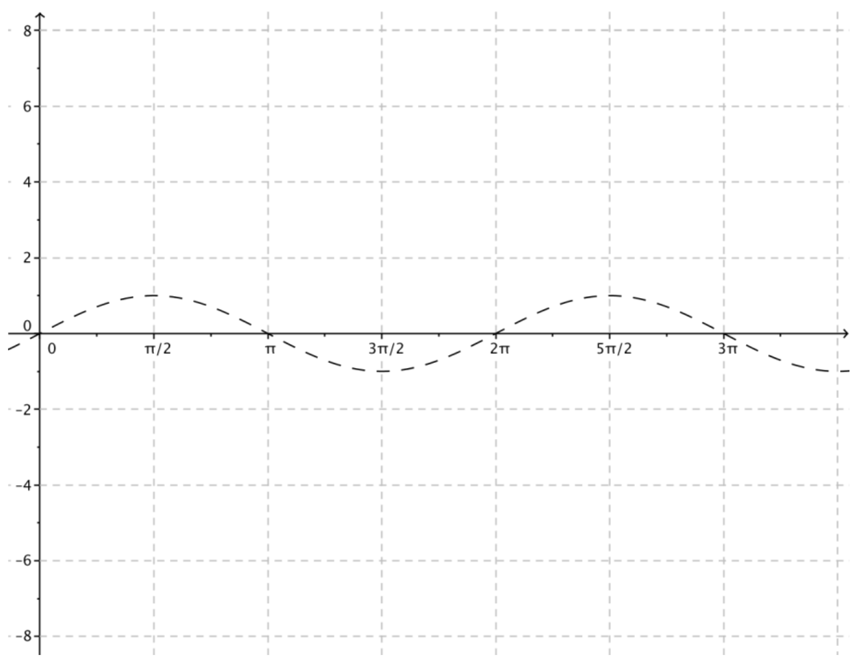
f. $g(x) = -\frac{2}{3}\sin(4x) - 3$



g. $g(x) = \pi \sin\left(\frac{x}{2}\right) + \pi$



h. $g(x) = 4 \sin\left(\frac{1}{2}(x - 5\pi)\right)$



Lesson Summary

This lesson investigated the effects of the parameters A, ω, h, and k on the graph of the function

$f(x) = A\sin\big(\omega(x-h)\big) + k$.

- The amplitude of the function is $|A|$; the vertical distance from a maximum point to the midline of the graph is $|A|$.
- The frequency of the function is $f = \frac{|\omega|}{2\pi}$, and the period is $P = \frac{2\pi}{|\omega|}$. The period is the vertical distance between two consecutive maximal points on the graph of the function.
- The phase shift is h. The value of h determines the horizontal translation of the graph from the graph of the sine function. If $h > 0$, the graph is translated h units to the right, and if $h < 0$, the graph is translated h units to the left.
- The graph of $y = k$ is the midline. The value of k determines the vertical translation of the graph compared to the graph of the sine function. If $k > 0$, then the graph shifts k units upward. If $k < 0$, then the graph shifts k units downward.

These parameters affect the graph of $f(x) = A\cos\big(\omega(x-h)\big) + k$ similarly.

Problem Set

1. For each function, indicate the amplitude, frequency, period, phase shift, horizontal and vertical translations, and equation of the midline. Graph the function together with a graph of the sine function $f(x) = \sin(x)$ on the same axes. Graph at least one full period of each function. No calculators are allowed.
   a. $g(x) = 3\sin\left(x - \frac{\pi}{4}\right)$
   b. $g(x) = 5\sin(4x)$
   c. $g(x) = 4\sin\left(3\left(x + \frac{\pi}{2}\right)\right)$
   d. $g(x) = 6\sin(2x + 3\pi)$ (Hint: First, rewrite the function in the form $g(x) = A\sin\big(\omega(x-h)\big)$.)

2. For each function, indicate the amplitude, frequency, period, phase shift, horizontal and vertical translations, and equation of the midline. Graph the function together with a graph of the sine function $f(x) = \cos(x)$ on the same axes. Graph at least one full period of each function. No calculators are allowed.
   a. $g(x) = \cos(3x)$
   b. $g(x) = \cos\left(x - \frac{3\pi}{4}\right)$
   c. $g(x) = 3\cos\left(\frac{x}{4}\right)$
   d. $g(x) = 3\cos(2x) - 4$
   e. $g(x) = 4\cos\left(\frac{\pi}{4} - 2x\right)$ (Hint: First, rewrite the function in the form $g(x) = A\cos\big(\omega(x-h)\big)$.)

3. For each problem, sketch the graph of the pairs of indicated functions on the same set of axes without using a calculator or other graphing technology.
   a. $f(x) = \sin(4x)$, $g(x) = \sin(4x) + 2$
   b. $f(x) = \sin\left(\frac{1}{2}x\right)$, $g(x) = 3\sin\left(\frac{1}{2}x\right)$
   c. $f(x) = \sin(-2x)$, $g(x) = \sin(-2x) - 3$
   d. $f(x) = 3\sin(x)$, $g(x) = 3\sin\left(x - \frac{\pi}{2}\right)$
   e. $f(x) = -4\sin(x)$, $g(x) = -4\sin\left(\frac{1}{3}x\right)$
   f. $f(x) = \frac{3}{4}\sin(x)$, $g(x) = \frac{3}{4}\sin(x - 1)$
   g. $f(x) = \sin(2x)$, $g(x) = \sin\left(2\left(x - \frac{\pi}{6}\right)\right)$
   h. $f(x) = 4\sin(x) - 3$, $g(x) = 4\sin\left(x - \frac{\pi}{4}\right) - 3$

Extension:

4. Show that if the graphs of the functions $f(x) = A\sin(\omega(x - h_1)) + k$ and $g(x) = A\sin(\omega(x - h_2)) + k$ are the same, then h_1 and h_2 differ by an integer multiple of the period.

5. Show that if h_1 and h_2 differ by an integer multiple of the period, then the graphs of $f(x) = A\sin(\omega(x - h_1)) + k$ and $g(x) = A\sin(\omega(x - h_2)) + k$ are the same graph.

6. Find the x-intercepts of the graph of the function $f(x) = A\sin(\omega(x - h))$ in terms of the period P, where $\omega > 0$.