Lesson 15: What Is a Trigonometric Identity?

Classwork

Exercises 1–3

1. Recall the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, where θ is any real number.

   a. Find $\sin(x)$, given $\cos(x) = \frac{3}{5}$, for $-\frac{\pi}{2} < x < 0$.

   b. Find $\tan(y)$, given $\cos(y) = -\frac{5}{13}$, for $\frac{\pi}{2} < y < \pi$.

   c. Write $\tan(z)$ in terms of $\cos(z)$, for $\pi < z < \frac{3\pi}{2}$.

2. Use the Pythagorean identity to do the following:
   a. Rewrite the expression $\cos(\theta)\sin^2(\theta) - \cos(\theta)$ in terms of a single trigonometric function. State the resulting identity.

   b. Rewrite the expression $(1 - \cos^2(\theta))\csc(\theta)$ in terms of a single trigonometric function. State the resulting identity.

   c. Find all solutions to the equation $2\sin^2(\theta) = 2 + \cos(\theta)$ in the interval $(0, 2\pi)$. Draw a unit circle that shows the solutions.

3. Which of the following statements are identities? If a statement is an identity, specify the values of x where the equation holds.

   a. $\sin(x + 2\pi) = \sin(x)$ where the functions on both sides are defined.

   b. $\sec(x) = 1$ where the functions on both sides are defined.

   c. $\sin(-x) = \sin(x)$ where the functions on both sides are defined.

   d. $1 + \tan^2(x) = \sec^2(x)$ where the functions on both sides are defined.

   e. $\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$ where the functions on both sides are defined.

   f. $\sin^2(x) = \tan^2(x)$ for all real x.

Lesson Summary

The Pythagorean identity: $\sin^2(\theta) + \cos^2(\theta) = 1$ for all real numbers θ.

Problem Set

1. Which of the following statements are trigonometric identities? Graph the functions on each side of the equation.
   a. $\tan(x) = \frac{\sin(x)}{\cos(x)}$ where the functions on both sides are defined.
   b. $\cos^2(x) = 1 + \sin(x)$ where the functions on both sides are defined.
   c. $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$ where the functions on both sides are defined.

2. Determine the domain of the following trigonometric identities:
   a. $\cot(x) = \frac{\cos(x)}{\sin(x)}$ where the functions on both sides are defined.
   b. $\cos(-u) = \cos(u)$ where the functions on both sides are defined.
   c. $\sec(y) = \frac{1}{\cos(y)}$ where the functions on both sides are defined.

3. Rewrite $\sin(x)\cos^2(x) - \sin(x)$ as an expression containing a single term.

4. Suppose $0 < \theta < \frac{\pi}{2}$ and $\sin(\theta) = \frac{1}{\sqrt{3}}$. What is the value of $\cos(\theta)$?

5. If $\cos(\theta) = -\frac{1}{\sqrt{5}}$, what are possible values of $\sin(\theta)$?

6. Use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, where θ is any real number, to find the following:
   a. $\cos(\theta)$, given $\sin(\theta) = \frac{5}{13}$, for $\frac{\pi}{2} < \theta < \pi$.
   b. $\tan(x)$, given $\cos(x) = -\frac{1}{\sqrt{2}}$, for $\pi < x < \frac{3\pi}{2}$.

7. The three identities below are all called Pythagorean identities. The second and third follow from the first, as you saw in Example 1 and the Exit Ticket.
   a. For which values of θ are each of these identities defined?
      i. $\sin^2(\theta) + \cos^2(\theta) = 1$, where the functions on both sides are defined.
      ii. $\tan^2(\theta) + 1 = \sec^2(\theta)$, where the functions on both sides are defined.
      iii. $1 + \cot^2(\theta) = \csc^2(\theta)$, where the functions on both sides are defined.
   b. For which of the three identities is 0 in the domain of validity?
   c. For which of the three identities is $\frac{\pi}{2}$ in the domain of validity?
   d. For which of the three identities is $-\frac{\pi}{4}$ in the domain of validity?