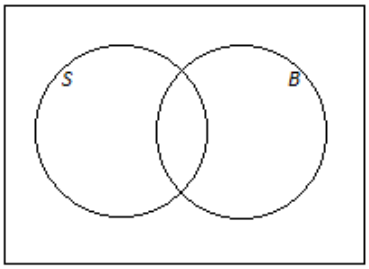
Lesson 5: Events and Venn Diagrams

Classwork

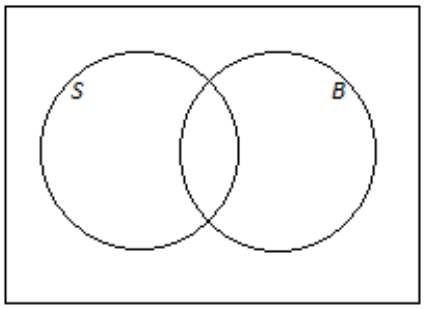
Example 1: Shading Regions of a Venn Diagram

At a high school, some students play soccer, and some do not. Also, some students play basketball, and some do not. This scenario can be represented by a Venn diagram, as shown below. The circle labeled S represents the students who play soccer, the circle labeled B represents the students who play basketball, and the rectangle represents all the students at the school.

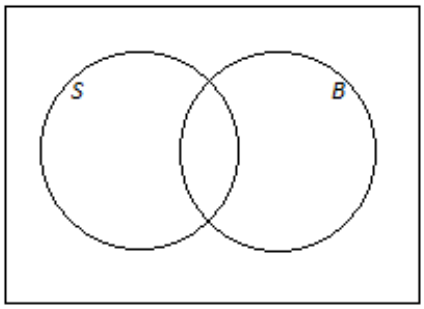


On the Venn diagrams provided, shade the region representing the following instances:

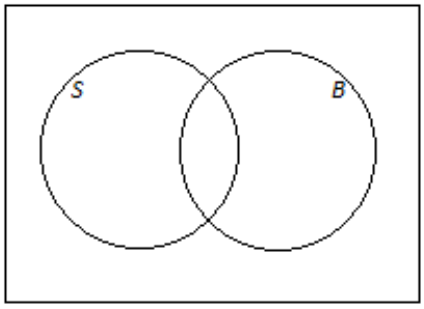
a. The students who play soccer



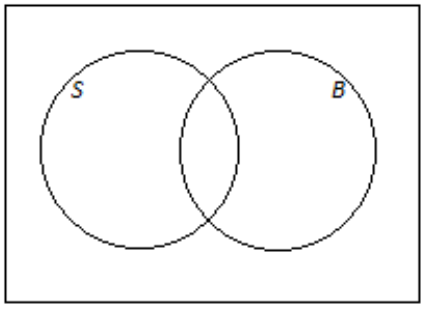
b. The students who do not play soccer



c. The students who play soccer and basketball



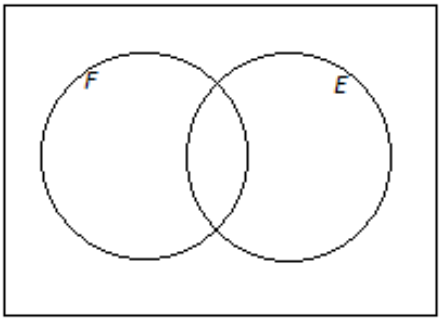
d. The students who play soccer or basketball



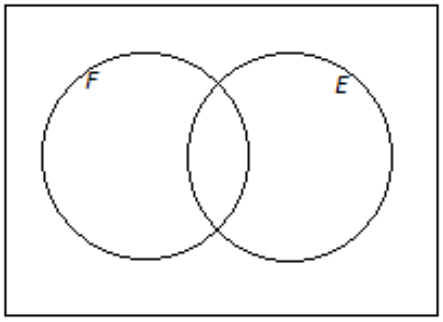
Exercise 1

An online bookstore offers a large selection of books. Some of the books are works of fiction, and some are not. Also, some of the books are available as e-books, and some are not. Let F be the set of books that are works of fiction, and let E be the set of books that are available as e-books. On the Venn diagrams provided, shade the regions representing the following instances:

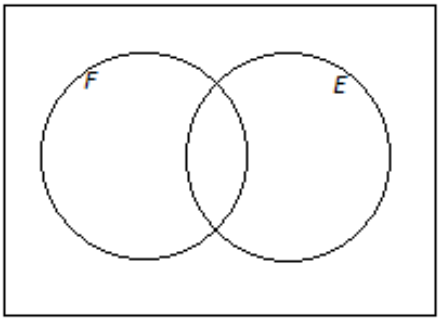
a. Books that are available as e-books



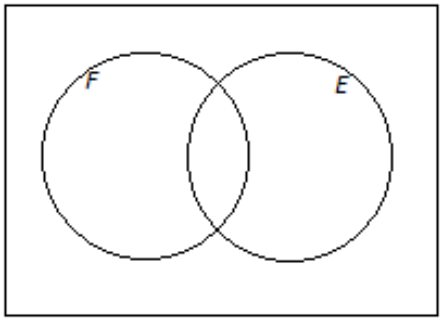
b. Books that are not works of fiction



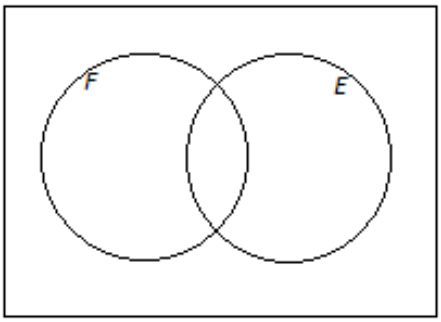
c. Books that are works of fiction and available as e-books



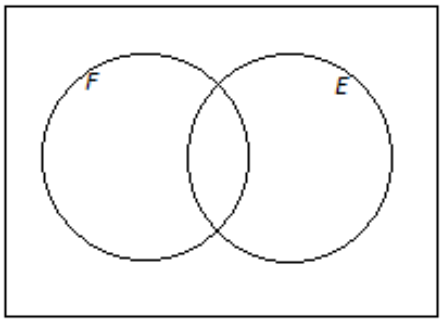
d. Books that are works of fiction or available as e-books



e. Books that are neither works of fiction nor available as e-books



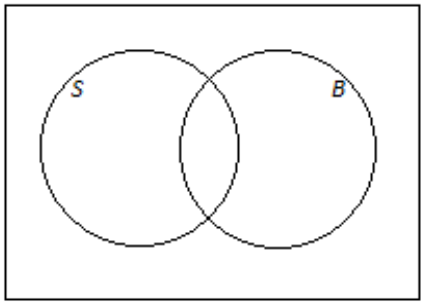
f. Books that are works of fiction that are not available as e-books



Example 2: Showing Numbers of Possible Outcomes (and Probabilities) in a Venn Diagram

Think again about the school introduced in Example 1. Suppose that 230 students play soccer, 190 students play basketball, and 60 students play both sports. There are a total of 500 students at the school.

a. Complete the Venn diagram below by writing the numbers of students in the various regions of the diagram.

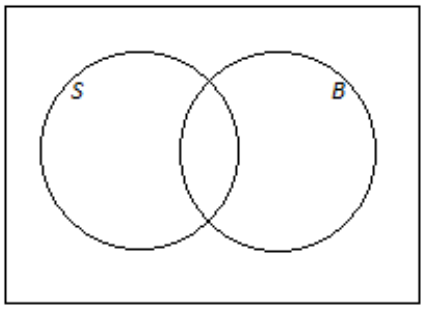


b. How many students play basketball but not soccer?

c. Suppose that a student will be selected at random from the school.

i. What is the probability that the selected student plays both sports?

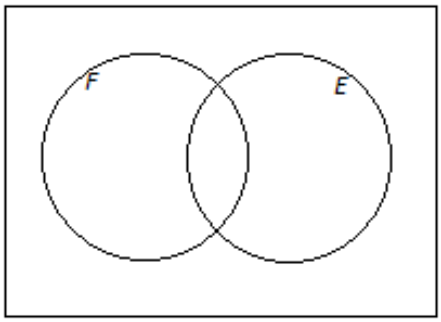
ii. Complete the Venn diagram below by writing the probabilities associated with the various regions of the diagram.



Example 3: Adding and Subtracting Probabilities

Think again about the online bookstore introduced in Exercise 1, and suppose that 62% of the books are works of fiction, 47% are available as e-books, and 14% are available as e-books but are not works of fiction. A book will be selected at random.

a. Using a Venn diagram, find the following probabilities:



i. The book is a work of fiction and available as an e-book.

ii. The book is neither a work of fiction nor available as an e-book.

b. Return to the information given at the beginning of the question: 62% of the books are works of fiction, 47% are available as e-books, and 14% are available as e-books but are not works of fiction.

i. How would this information be shown in a hypothetical 1000 table? (Show your answers in the table provided below.)

	Fiction	Not Fiction	Total
Available as E-Book			
Not Available as E-Book			
Total			1,000

ii. Complete the hypothetical 1000 table given above.

iii. Complete the table below showing the probabilities of the events represented by the cells in the table.

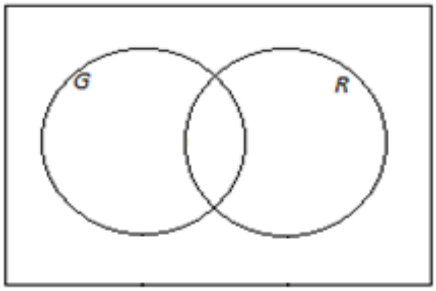
	Fiction	Not Fiction	Total
Available as E-Book			
Not Available as E-Book			
Total			

iv. How do the probabilities in your table relate to the probabilities you calculated in part (a)?

Exercise 2

When a fish is selected at random from a tank, the probability that it has a green tail is 0.64, the probability that it has red fins is 0.25, and the probability that it has both a green tail and red fins is 0.19.

a. Draw a Venn diagram to represent this information.



b. Find the following probabilities:

i. The fish has red fins but does not have a green tail.

ii. The fish has a green tail but not red fins.

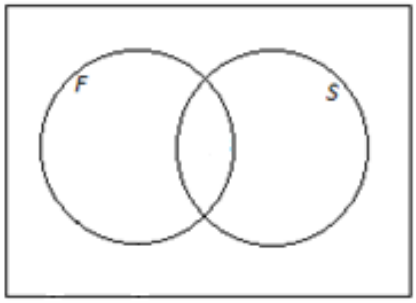
iii. The fish has neither a green tail nor red fins.

c. Complete the table below showing the probabilities of the events corresponding to the cells of the table.

	Green Tail	Not Green Tail	Total
Red Fins			
Not Red Fins			
Total			

Exercise 3

In a company, 43% of the employees have access to a fax machine, 38% have access to a fax machine and a scanner, and 24% have access to neither a fax machine nor a scanner. Suppose that an employee will be selected at random. Using a Venn diagram, calculate the probability that the randomly selected employee will not have access to a scanner. (Note that Venn diagrams and probabilities use decimals or fractions, not percentages.) Explain how you used the Venn diagram to determine your answer.



Lesson Summary

In a probability experiment, the events can be represented by circles in a Venn diagram.

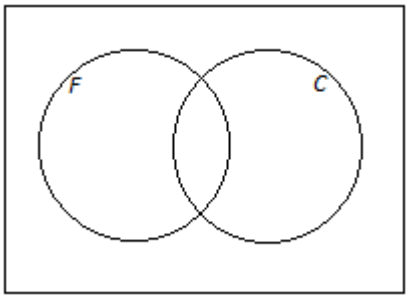
Combinations of events using *and*, *or*, and *not* can be shown by shading the appropriate regions of the Venn diagram.

The number of possible outcomes can be shown in each region of the Venn diagram; alternatively, probabilities may be shown. The number of outcomes in a given region (or the probability associated with it) can be calculated by adding or subtracting the known numbers of possible outcomes (or probabilities).

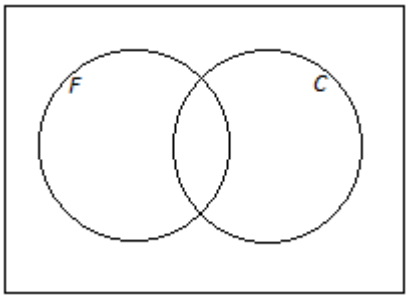
Problem Set

1. On a flight, some of the passengers have frequent-flier status, and some do not. Also, some of the passengers have checked baggage, and some do not. Let the set of passengers who have frequent-flier status be F and the set of passengers who have checked baggage be C. On the Venn diagrams provided, shade the regions representing the following instances:

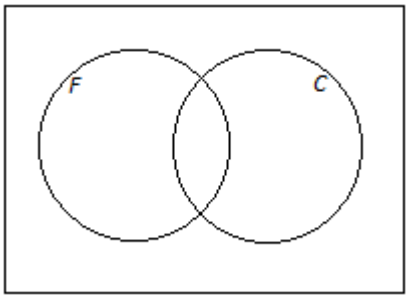
   a. Passengers who have frequent-flier status and have checked baggage

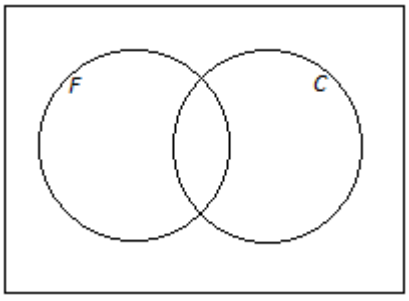
   b. Passengers who have frequent-flier status or have checked baggage

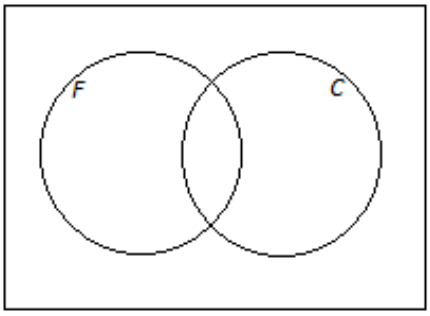
   c. Passengers who do not have both frequent-flier status and checked baggage

   d. Passengers who have frequent-flier status or do not have checked baggage

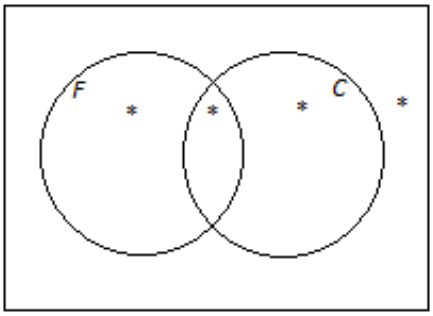
   

2. For the scenario introduced in Problem 1, suppose that, of the 400 people on the flight, 368 have checked baggage, 228 have checked baggage but do not have frequent-flier status, and 8 have neither frequent-flier status nor checked baggage.



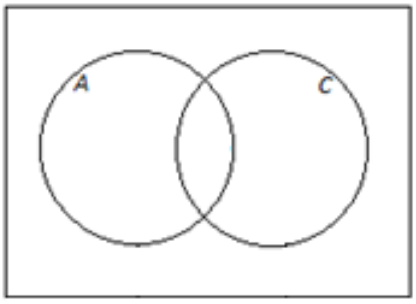
   a. Using a Venn diagram, calculate the following:
      i. The number of people on the flight who have frequent-flier status and have checked baggage
      ii. The number of people on the flight who have frequent-flier status

   b. In the Venn diagram provided below, write the probabilities of the events associated with the regions marked with a star (*).



3. When an animal is selected at random from those at a zoo, the probability that it is North American (meaning that its natural habitat is in the North American continent) is 0.65, the probability that it is both North American and a carnivore is 0.16, and the probability that it is neither American nor a carnivore is 0.17.

   a. Using a Venn diagram, calculate the probability that a randomly selected animal is a carnivore.



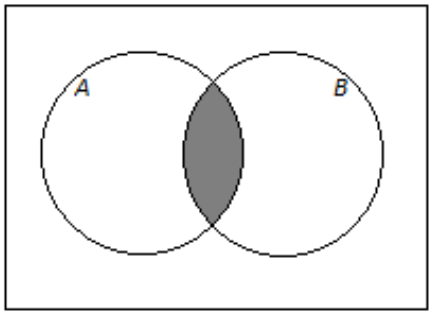
   b. Complete the table below showing the probabilities of the events corresponding to the cells of the table.

	North American	Not North American	Total
Carnivore			
Not Carnivore			
Total			

4. This question introduces the mathematical symbols for *and*, *or*, and *not*.

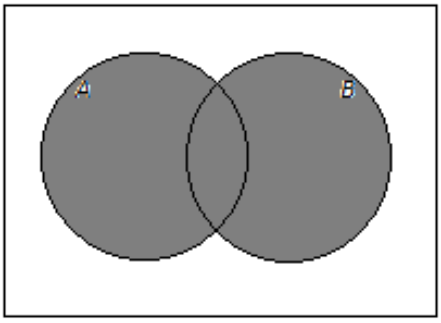
   Considering all the people in the world, let A be the set of Americans (citizens of the United States), and let B be the set of people who have brothers.

   - The set of people who are Americans and have brothers is represented by the shaded region in the Venn diagram below.



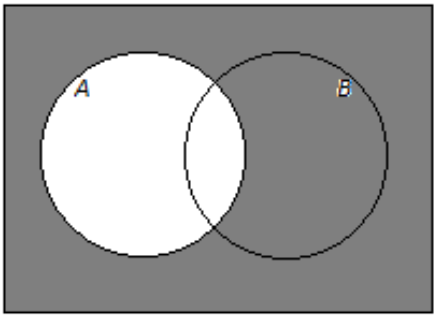
   This set is written $A \cap B$ (read A intersect B), and the probability that a randomly selected person is American and has a brother is written $P(A \cap B)$.

   - The set of people who are Americans or have brothers is represented by the shaded region in the Venn diagram below.



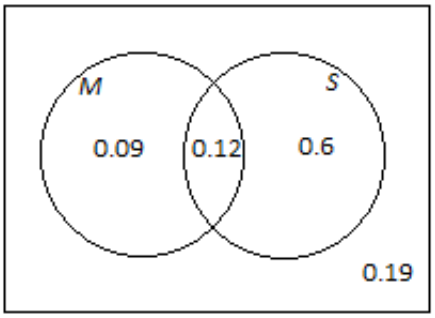
   This set is written $A \cup B$ (read A union B), and the probability that a randomly selected person is American or has a brother is written $P(A \cup B)$.

   - The set of people who are not Americans is represented by the shaded region in the Venn diagram below.



   This set is written A^C (read A complement), and the probability that a randomly selected person is not American is written $P(A^C)$.

Now, think about the cars available at a dealership. Suppose a car is selected at random from the cars at this dealership. Let the event that the car has manual transmission be denoted by M, and let the event that the car is a sedan be denoted by S. The Venn diagram below shows the probabilities associated with four of the regions of the diagram.



a. What is the value of $P(M \cap S)$?

b. Complete this sentence using *and* or *or*:

$P(M \cap S)$ is the probability that a randomly selected car has a manual transmission _____ is a sedan.

c. What is the value of $P(M \cup S)$?

d. Complete this sentence using *and* or *or*:

$P(M \cup S)$ is the probability that a randomly selected car has a manual transmission _____ is a sedan.

e. What is the value of $P(S^C)$?

f. Explain the meaning of $P(S^C)$.