



# **Lesson 23: Experiments and the Role of Random**

# **Assignment**

#### **Student Outcomes**

- Given a description of a statistical experiment, students identify the response variable and the treatments.
- Students recognize the different purposes of random selection and of random assignment.
- Students recognize the importance of random assignment in statistical experiments.

#### **Lesson Notes**

Experiments are introduced as investigations designed to compare the effect of two treatments on a response variable. This lesson revisits the distinction between random selection and random assignment and also explores the role of random assignment in carrying out a statistical experiment to compare two treatments.

#### Classwork

## Exercises 1-4 (8 minutes): Experiments

Read the information about the two studies to the class. Give students about two to three minutes to work with a partner to answer Exercise 1.

#### Exercises 1-4: Experiments

Two studies are described below. One is an observational study, while the other is an experiment.

#### Study A:

A new dog food, specially designed for older dogs, has been developed. A veterinarian wants to test this new food against another dog food currently on the market to see if it improves dogs' health. Thirty older dogs were randomly assigned to either the "new" food group or the "current" food group. After they were fed either the "new" or "current" food for six months, their improvement in health was rated.

#### Study B:

The administration at a large school wanted to determine if there was a difference in the mean number of text messages sent by ninth-grade students and by eleventh-grade students during a day. Students in a random sample of 30 ninth-grade students were asked how many text messages they sent per day. Students in another random sample of 30 eleventh-grade students were asked how many text messages they sent per day. The difference in the mean number of texts per day was determined.

## Scaffolding:

For struggling students, offer a graphic organizer that could be used to remind them of the characteristics of observational studies and experiments.

Also, consider using a visual depiction of the experiments.



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1. Which study is the experiment? Explain. Discuss the answer with your partner.

Study A is an experiment because treatments are assigned to the dogs, and a response is measured.



Now, let students try to form their own definitions for a *subject, response variable*, and *treatment group* by identifying what each one is in the experiment; allow students to work with their partners to answer Exercises 2–4. Convey the following to students before each exercise, and allow two minutes for students to respond to each question:

- We are going to introduce some specific vocabulary used in experiments. In the example above, the dogs are the subjects. Think about what happened in the experiment, and describe what a subject is in an experiment.
  - 2. In your own words, describe what a subject is in an experiment.

A subject is a participant in the experiment.

- The dogs' health was the response variable. Think about the experiment, and describe what a response variable is in an experiment.
  - 3. In your own words, describe what a response variable is in an experiment.

A response variable is not controlled by the experimenter and is measured as part of the experiment.

- "New" food and "old" food were the treatments. Describe what a treatment is in an experiment.
  - 4. In your own words, describe what a treatment is in an experiment.

A treatment is the condition(s) to which subjects are randomly assigned by the experimenter.

## Exercises 5-9 (10-13 minutes): Random Selection and Random Assignment



Read and discuss the differences between the terms *random selection* and *random assignment*. Have students work with a partner to answer Exercises 5 through 8. Verify that students understand the differences between random selection and random assignment. Allow students about five minutes to complete Exercise 9.

### Exercises 5-9: Random Selection and Random Assignment

Take another look at the two studies described above. Study A (the dog food study) is an experiment, while study B (text messages) is an observational study. The term *random sample* implies that a sample was randomly selected from a population. The terms *random selection* and *random assignment* have very different meanings.

Random selection refers to randomly selecting a sample from a population. Random selection allows generalization to a population and is used in well-designed observational studies. Sometimes, but not always, the subjects in an experiment are randomly selected.

Random assignment refers to randomly assigning the subjects in an experiment to treatments. Random assignment allows for cause-and-effect conclusions and is used in well-designed experiments.

## Scaffolding:

- The term assignment may be familiar to students (e.g., homework assignments), but in this context it may need explanation.
- In this lesson, assignment refers to allocating or assigning a subject to a treatment.
- Consider using a Frayer model diagram for some of the vocabulary in this lesson.



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In study B, the data were collected from two random samples of students.

5. Can the results of the survey be generalized to all ninth-grade and all eleventh-grade students at the school? Why or why not? Discuss the answer with your partner.

Yes, the results of the survey can be generalized to all ninth- and eleventh-grade students at the school because the students were randomly selected.

6. Suppose there really is a difference in the mean number of texts sent by ninth-grade students and by eleventh-grade students. Can we say that the grade level of the students is the cause of the difference in the mean number of texts sent? Why or why not? Discuss the answer with your partner.

No, we cannot say that the grade level of the students is the cause of the difference in the mean number of texts sent because observational studies do not allow for cause-and-effect conclusions.

In study A, the dogs were randomly assigned to one of the two types of food.

7. Suppose the dogs that were fed the new food showed improved health. Can we say that the new food is the cause of the improvement in the dogs' health? Why or why not? Discuss the answer with your partner.

Yes, we can say that the new food is the cause of improvement in the dogs' health because the dogs were randomly assigned to treatments. This allows for cause-and-effect conclusions.

8. Can the results of the dog food study be generalized to all dogs? To all older dogs? Why or why not? Discuss the answer with your partner.

No, the results cannot be generalized to all dogs or to all older dogs because the dogs were not randomly selected from the population of all dogs or from the population of older dogs.

The table below summarizes the differences between the terms random selection and random assignment.

9. For each statement, put a check mark in the appropriate column(s,) and explain your choices.

	Random Selection	Random Assignment
Used in Experiments	٧	√
Used in Observational Studies	√	
Allows Generalization to the Population	√	
Allows a Cause-and-Effect Conclusion		√

## Exercises 10-17 (22-25 minutes)

Students should work independently on this set of exercises. Read aloud and discuss the experiment. Be sure to address the concerns that the acres along the river and trees may have a richer nutrient level in the soil and may also get less sunshine than the acres farther away from the river and farther south of the trees. (An acre is  $43,560 \, \text{ft}^2$ , or approximately  $209 \, \text{ft}$ . An acre is almost the size of a football field.) Allow about two minutes for students to answer Exercise 10.



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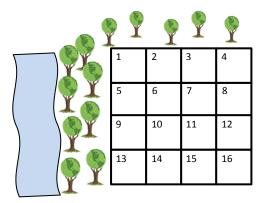
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#### Exercise 10

What is the purpose of random assignment in experiments? To answer this, consider the following investigation:

A researcher wants to determine if the yield of corn is different when the soil is treated with one of two different types of fertilizers, fertilizer A and fertilizer B. The researcher has 16 acres of land located beside a river that has several trees along its bank. There are also a few trees to the north of the 16 acres. The land has been divided into 16 one-acre plots. (See the diagram below.) These 16 plots are to be planted with the same type of corn but can be fertilized differently. At the end of the growing season, the corn yield will be measured for each plot, and the mean yields for the plots assigned to each fertilizer will be compared.



- 10. For the experiment, identify the following, and explain each answer:
  - a. Subjects (Hint: not always people or animals)

The 16 plots of land are the subjects because that is what the experiment is being carried out on.

b. Treatments

Fertilizer A and fertilizer B are the treatments because that is what is different in the experiment.

c. Response variable

The yield of corn is the response variable because that is what is being measured after treatment.

Next, students need to randomly assign each plot to one of the two treatments, fertilizer A or fertilizer B. Below are instructions for two different methods for the randomization process. Either method is appropriate. Students should develop a plan for randomly assigning the plots, share the responses, discuss, and then choose their preferred method. Consider doing both, starting with the paper method. Allow students 8–10 minutes to carry out the randomization method and answer Exercise 11.





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## **Paper Method**

- 1. Give each student 16 precut slips of paper (approximately a 2-inch square).
- 2. Have students write a number from 1 to 16 on each square.
- 3. Turn the squares upside down. Mix well.
- 4. Separate the squares into 2 piles of 8, identifying one pile as A and the other pile as B.
- 5. Turn the slips of paper over.
- 6. The numbers in pile A correspond to the plots that receive the fertilizer A treatment.
- 7. The numbers in pile B correspond to the plots that receive the fertilizer B treatment.
- 8. Write the letters A and B in the corresponding squares in the diagram. (Exercise 11)

#### **Calculator Method**

- 1. When using the random number generator in a graphing calculator, the random generator function on each calculator often needs to be seeded with a unique number.\* For the TI-83 or TI-84 calculator, have each student type in a unique number, such as his student ID number. Press the Store, or STO>, key. Go to the Probability, or PRB, menu. This menu is accessed by pressing the MATH key. Select 1: rand. Press Enter to select. Press Enter to perform the command.
- 2. Next, use the command to generate random integers from 1 to 16. For the TI-83 or TI-84 calculator, go to the Probability, PRB, menu. This menu is accessed by pressing the MATH key. Select 8: randIntNoRep. In the parentheses, type 1,16. Press Enter to do the command.
- 3. The first 8 unique numbers generated represent the plots of land that are assigned to fertilizer A.
- 4. The remaining 8 numbers represent the plots of land that are assigned to fertilizer B.
- 5. Write the letters A and B in the corresponding squares in the diagram. (Exercise 11)

\*The random generator function on a calculator is an algorithm (a program) that produces pseudo random numbers — numbers that behave like random numbers. This program often has a default value that it uses to generate numbers. If this default value is not changed, the generator is not seeded; then, each calculator generates the same set of numbers.

### Exercise 11

Next, you need to assign the plots to one of the two treatments. To do this, follow the instructions given by your teacher.

11. Write A (for fertilizer A) or B (for fertilizer B) in each of the 16 squares in the diagram so that it corresponds to your random assignment of fertilizer to plots.

Answers will vary. A sample response is provided.

1	2	3	4
В	Α	Α	В
5	6	7	8
В	Α	В	Α
9	10	11	12
Α	Α	В	Α
13	14	15	16
Α	В	В	В

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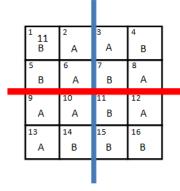


After students have completed the table, have them compare their allocations of the two treatments. Be sure to stress that random assignment produces a large variety of possible outcomes. Next, students investigate their results. Students are investigating locations with respect to central vertical (blue) and horizontal (red) lines. Students see that their chosen random distributions gave a good spread of fertilizer A and B above and below the red line and to the right and left of the blue line. Allow five to eight minutes for students to answer Exercises 12–15.

### Exercises 12-15

Let's investigate the results of the random assignment of the fertilizer types to the plots.

A sample response is provided.



12. On the diagram above, draw a vertical line down the center of the 16 plots of land.

See the chart above.

13. Count the number of plots on the left side of the vertical line that will receive fertilizer A. Count the number of plots on the right side of the vertical line that will receive fertilizer A.

There are 5 plots on the left and 3 plots on the right. Most students get results that have an approximately equal split. A few students may get 6 for one side of the line and 2 for the other side. Point out that this is possible with random assignment but not very likely.

14. On the diagram above, draw a horizontal line through the center of the 16 plots of land.

See the chart above.

 Count the number of plots above the horizontal line that will receive fertilizer A. Count the number of plots below the horizontal line that will receive fertilizer A.

There are 4 plots above the line and 4 plots below the line. Most students get results that have an approximately equal split. A few students may get 6 for one side of the line and 2 for the other side. Point out that this is possible with random assignment but not very likely.

Above	Below
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When students have completed Exercises 12–15, ask them to raise their hands if their randomization method produced an approximately even split left of the vertical line versus right of the vertical line. Comment on the fact that most students raised their hands.

Next, ask students to raise their hands if their randomization methods produced an approximately even split above the horizontal line versus below the horizontal line. Comment on the fact that most students raised their hands.

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Lesson 23:

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Then, hold a class discussion about the proximity of the river and of the northern trees. Stress the following ideas:

- About half of the plots of land close to the river received fertilizer A, while the other half received fertilizer B.
- About half of the plots of land close to the northern trees received fertilizer A, while the other half received fertilizer B.

Allow students five to eight minutes to answer Exercises 16 and 17.

#### Exercises 16-17

In experiments, random assignment is used as a way of ensuring that the groups that receive each treatment are as much alike as possible with respect to other factors that might affect the response.

16. Explain what this means in the context of this experiment.

This means that approximately half of the plots that are close to the river were assigned fertilizer A, and the remaining half were assigned fertilizer B. Also, approximately half of the plots that are close to the northern trees were assigned fertilizer A, and the remaining half were assigned fertilizer B.

17. Suppose that, at the end of the experiment, the mean yield for one of the fertilizers is quite a bit higher than the mean yield for the other fertilizer. Explain why it would be reasonable to say that the type of fertilizer is the cause of the difference in yield and not the proximity to the river or to the northern trees.

Because about half the plots of land close to the river receive fertilizer A and the other half receive fertilizer B, we can see how the fertilizer works regardless of its proximity to the river. Also, because about half of the plots of land close to the northern trees receive fertilizer A and the other half receive fertilizer B, we can see how the fertilizer works regardless of its proximity to the northern trees.

## Closing (2 minutes)

Ask students to summarize the main ideas of the lesson in writing or with a neighbor. Use this as an opportunity to informally assess comprehension of the lesson. The Lesson Summary below offers some important ideas that should be included.

## **Lesson Summary**

- An experiment is an investigation designed to compare the effect of two or more treatments on a response variable.
- A subject is a participant in the experiment.
- The response variable is a variable that is not controlled by the experimenter and that is measured as part of the experiment.
- The treatments are the conditions to which subjects are randomly assigned by the experimenter.
  - Random selection refers to randomly selecting a sample from a population.
    - Random selection allows for generalization to a population.
- Random assignment refers to randomly assigning subjects to treatment groups.
  - Random assignment allows for cause-and-effect conclusions.
  - The purpose of random assignment in an experiment is to create similar groups of subjects for each of the treatments in the experiment.

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Exit Ticket (2 minutes)



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Lesson 23



ALGEBRA II



Name \_\_\_\_\_ Date \_\_\_\_

# Lesson 23: Experiments and the Role of Random Assignment

## **Exit Ticket**

Runners who suffered from shin splints were randomly assigned to one of two stretching routines. One of the routines involved a series of pre-run and post-run dynamic stretches that last approximately 5 minutes before and after the run. The other routine involved a 1-minute hamstring stretch pre-run and no stretching post-run. After a 45-minute run, each runner will be assessed for shin splints.

a. Explain why this is an experiment.

- b. Identify the subjects.
- c. Identify the treatments.

- d. Identify the response variable.
- e. Why are the runners randomly assigned to one of two stretching routines?



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## **Exit Ticket Sample Solutions**

Runners who suffered from shin splints were randomly assigned to one of two stretching routines. One of the routines involved a series of pre-run and post-run dynamic stretches that last approximately 5 minutes before and after the run. The other routine involved a 1-minute hamstring stretch pre-run and no stretching post-run. After a 45-minute run, each runner will be assessed for shin splints.

Explain why this is an experiment.

It is an investigation that is designed to compare the effect of two treatments to a response variable.

b. Identify the subjects.

The subjects are runners.

c. Identify the treatments.

The treatments are dynamic pre-run and post-run stretching routine and the 1-minute hamstring stretch.

d. Identify the response variable.

The response variable is the incidence of shin splints.

e. Why are the runners randomly assigned to one of two stretching routines?

The experimenter wants to create similar groups of runners for each of the treatments (stretching routines) in the experiment, which would allow for a cause-and-effect conclusion.

## **Problem Set Sample Solutions**

For Problems 1 through 5, identify (i) the subjects, (ii) the treatments, and (iii) the response variable for each experiment.

- A botanist was interested in determining the effects of watering (three days a week or daily) on the heat rating of
  jalapeño peppers. The botanist wanted to know which watering schedule would produce the highest heat rating in
  the peppers. He conducted an experiment, randomly assigning each watering schedule to half of 12 plots that had
  similar soil and full sun. The average final heat rating for the peppers grown in each plot was recorded at the end of
  the growing season.
  - i. Plots of peppers
  - ii. Watering three days a week and watering daily
  - iii. Heat rating
- 2. A manufacturer advertises that its new plastic cake pan bakes cakes more evenly. A consumer group wants to carry out an experiment to see if the plastic cake pans do bake more evenly than standard metal cake pans. Twenty cake mixes (same brand and type) are randomly assigned to either the plastic pan or the metal pan. All of the cakes are baked in the same oven. The rating scale was then used to rate the evenness of each cake.
  - i. Cake mixes
  - ii. Plastic pan and metal pan

Lesson 23:

iii. Evenness rating



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- 3. The city council of a large city is considering a new law that prohibits talking on a cell phone while driving. A consumer rights organization wants to know if talking on a cell phone while driving distracts a person's attention, causing that person to make errors while driving. An experiment is designed that uses a driving simulator to compare the two treatments: driving while talking on a cell phone and driving while not talking on a cell phone. The number of errors made while driving on an obstacle course will be recorded for each driver. Each person in a random sample of 200 licensed drivers in the city was asked to participate in the experiment. All of the drivers agreed to participate in the experiment. Half of the drivers were randomly assigned to drive an obstacle course while talking on a phone. The remaining half were assigned to drive the obstacle course while not talking on a phone.
  - i. The 200 randomly selected drivers who took part in the experiment
  - ii. Driving while talking on a cell phone and driving while not talking on a cell phone
  - iii. Number of errors made while driving on an obstacle course
- 4. Researchers studied 208 infants whose brains were temporarily deprived of oxygen as a result of complications at birth (*The New England Journal of Medicine*, October 13, 2005). An experiment was performed to determine if reducing body temperature for three days after birth improved their chances of surviving without brain damage. Infants were randomly assigned to usual care or whole-body cooling. The amount of brain damage was measured for each infant.
  - i. Infants born with a temporary deprivation of oxygen at birth
  - ii. Usual care or whole-body cooling
  - iii. Amount of brain damage
- The head of the quality control department at a printing company would like to carry out an experiment to determine which of three different glues results in the greatest binding strength. Copies of a book were randomly assigned to one of the three different glues.
  - i. Copies of a book
  - ii. Three different glues
  - iii. Binding strength
- 6. In Problem 3, suppose that drivers who talked on the phone while driving committed more errors on the obstacle course than drivers who did not talk on the phone while driving. Can we say that talking on the cell phone while driving is the cause of the increased errors on the obstacle course? Why or why not?

Yes, we can say that talking on the cell phone while driving is the cause of the increased errors on the obstacle course because the drivers were randomly assigned to one of two treatments.

- 7. Can the results of the experiment in Problem 3 be generalized to all licensed drivers in the city? Why or why not?
  - Yes, the results can be generalized to all licensed drivers in the city because the drivers were randomly selected from licensed drivers in the city.
- 8. In Problem 4, one of the treatment groups was to use usual care for the infants. Why was this treatment group included in the experiment?

The usual care treatment group is used as a comparison for how well the cooling treatment worked.

9. In Problem 5, why were copies of only one book used in the experiment?

This prevents other factors, such as number of pages, from interfering with the results of the experiment.



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