Lesson 29: Solving Radical Equations

Student Outcomes

- Students develop facility in solving radical equations.

Lesson Notes

In the previous lesson, students were introduced to the notion of solving radical equations and checking for extraneous solutions (**A-REI.A.2**). Students continue this work by looking at radical equations that contain variables on both sides. The main point to stress to students is that radical equations become polynomial equations through exponentiation. So we really have not left the notion of polynomials that have been studied throughout this module. This lesson also provides opportunities to emphasize MP.7 (look for and make use of structure).

Classwork

Discussion (5 minutes)

Before beginning the lesson, remind students of past experiences by providing the following scenario, which illustrates a case when an operation performed to both sides of an equation has changed the set of solutions.

Carlos and Andrea were solving the equation $x^2 + 2x = 0$. Andrea says that there are two solutions, 0 and -2. Carlos says the only solution is -2 because he divided both sides by x and got $x + 2 = 0$. Who is correct and why?

- Do both 0 and -2 satisfy the original equation?
  - *Yes. If we replace x with either 0 or -2, the answer is 0.*
- What happened when Carlos divided both sides of the equation by x?
  - *He changed the solutions from 0 and -2 to simply -2. He lost one solution to the equation.*
- What does this say about the solution of equations after we have performed algebraic operations on both sides?
  - *Performing algebraic steps may alter the set of solutions to the original equation.*

Now, Carlos and Andrea are solving the equation $\sqrt{x} = -3$. Andrea says the solution is 9 because she squared both sides and got $x = 9$. Carlos says there is no solution. Who is correct? Why?

MP.3

- Was Andrea correct to square both sides?
  - *Yes. To eliminate a radical from an equation, we raise both sides to an exponent.*
- Is she correct that the solution is 9?
  - *No. Carlos is correct. If we let $x = 9$, then we get $\sqrt{9} = 3$, and $3 \neq -3$, so 9 is not a solution.*

Scaffolding

- Use several examples to illustrate that if $a > 0$, then an equation of the form $\sqrt{x} = -a$ will not have a solution (e.g., $\sqrt{x} = -4$, $\sqrt{x} = -5$).
- Extension: Write an equation that has an extraneous solution of $x = 50$.

- What is the danger in squaring both sides of an equation?
  - *It sometimes produces an equation whose solution set is not equivalent to that of the original equation. If both sides of* $\sqrt{x} = -3$ *are squared, the equation* $x = 9$ *is produced, but* 9 *is not a solution to the original equation. The original equation has no solution.*
- Because of this danger, what is the final essential step of solving a radical equation?
  - *Checking the solution or solutions to ensure that an extraneous solution was not produced by the step of squaring both sides.*
- How could we have predicted that the equation would have no solution?
  - *The square root of a number is never equal to a negative value, so there is no* x*-value so that* $\sqrt{x} = -3$.

Example 1 (5 minutes)

MP.1 While this problem is difficult, students should attempt to solve it on their own first, by applying their understandings of radicals. Students should be asked to verify the solution they come up with and describe their solution method. Discuss Example 1 as a class once they have worked on it individually.

Example 1

Solve the equation $6 = x + \sqrt{x}$.

$$6 - x = \sqrt{x}$$
$$(6 - x)^2 = \sqrt{x}^2$$
$$36 - 12x + x^2 = x$$
$$x^2 - 13x + 36 = 0$$
$$(x - 9)(x - 4) = 0$$

The solutions are 9 ***and*** 4.

Check $x = 9$:

$$9 + \sqrt{9} = 9 + 3 = 12$$
$$6 \neq 12$$

Check $x = 4$:

$$4 + \sqrt{4} = 4 + 2 = 6$$

So, 9 ***is an extraneous solution.***

The only valid solution is 4.

- How does this equation differ from the ones from yesterday's lesson?
  - *There are two* x's*; one inside and one outside of the radical.*
- Explain how you were able to determine the solution to the equation above.
  - *Isolate the radical and square both sides. Solve the resulting equation.*
- Did that change the way in which the equation was solved?
  - *Not really; we still eliminated the radical by squaring both sides.*
- What type of equation were we left with after squaring both sides?
  - *A quadratic polynomial equation*
- Why did 9 fail to work as a solution?
  - *The square root of* 9 *takes only the positive value of* 3.

Exercises 1–4 (13 minutes)

Allow students time to work the problems independently and then pair up to compare solutions. Use this time to informally assess student understanding by examining their work. Display student responses, making sure that students checked for extraneous solutions.

Exercises 1–4

Solve.

1. $3x = 1 + 2\sqrt{x}$

   The only solution is 1.

   Note that $\frac{1}{9}$ *is an extraneous solution.*

2. $3 = 4\sqrt{x} - x$

   The two solutions are 9 *and* 1.

3. $\sqrt{x+5} = x - 1$

   The only solution is 4.

   Note that -1 *is an extraneous solution.*

4. $\sqrt{3x+7} + 2\sqrt{x-8} = 0$

   There are no solutions.

- When solving Exercise 1, what solutions did you find? What happened when you checked these solutions?
  - *The solutions found were* $\frac{1}{9}$ *and* 1. *Only* 1 *satisfies the original equation, so* $\frac{1}{9}$ *is an extraneous solution.*
- Did Exercise 2 have any extraneous solutions?
  - *No. Both solutions satisfied the original equation.*

MP.7

- Looking at Exercise 4, could we have predicted that there would be no solution?
  - *Yes. The only way the two square roots could add to zero would be if both of them produced a zero, meaning that* $3x + 7 = 0$ *and* $x - 8 = 0$. *Since* x *cannot be both* $-\frac{7}{3}$ *and* 8, *both radicals cannot be simultaneously zero. Thus, at least one of the square roots will be positive, and they cannot sum to zero.*

Example 2 (5 minutes)

MP.7

What do we do when there is no way to isolate the radical? What is going to be the easiest way to square both sides? Give students time to work on Example 2 independently. Point out that even though we had to square both sides twice, we were still able to rewrite the equation as a polynomial.

> *Scaffolding:*
>
> What if we had squared both sides of the equation as it was presented? Have early finishers work out the solution this way and share with the class.

Example 2

Solve the equation $\sqrt{x} + \sqrt{x+3} = 3$.

$$\sqrt{x+3} = 3 - \sqrt{x}$$
$$\left(\sqrt{x+3}\right)^2 = (3 - \sqrt{x})^2$$
$$x + 3 = 9 - 6\sqrt{x} + x$$
$$1 = \sqrt{x}$$
$$1 = x$$

Check:

$$\sqrt{1} + \sqrt{1+3} = 1 + 2 = 3$$

So the solution is 1.

Exercises 5–6 (7 minutes)

Allow students time to work the problems independently and then pair up to compare solutions. Circulate to assess understanding. Consider targeted instruction with a small group of students while others work independently. Display student responses, making sure that students check for extraneous solutions.

Exercises 5–6

Solve the following equations.

5. $\sqrt{x-3}+\sqrt{x+5}=4$

4

6. $3+\sqrt{x}=\sqrt{x+81}$

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Closing (5 minutes)

Ask students to respond to these prompts in writing or with a partner. Use these responses to informally assess their understanding of the lesson.

- How did these equations differ from the equations seen in the previous lesson?
  - *Most of them contained variables on both sides of the equation or a variable outside of the radical.*
- How were they similar to the equations from the previous lesson?
  - *They were solved using the same process of squaring both sides. Even though they were more complicated, the equations could still be rewritten as a polynomial equation and solved using the same process seen throughout this module.*
- Give an example where $a^n = b^n$ but $a \neq b$.
  - *We know that* $(-3)^2 = 3^2$ *but* $-3 \neq 3$.

Lesson Summary

If $a = b$ and n is an integer, then $a^n = b^n$. However, the converse is not necessarily true. The statement $a^n = b^n$ does not imply that $a = b$. Therefore, it is necessary to check for extraneous solutions when both sides of an equation are raised to an exponent.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 29: Solving Radical Equations

Exit Ticket

1. Solve $\sqrt{2x + 15} = x + 6$. Verify the solution(s).

2. Explain why it is necessary to check the solutions to a radical equation.

Exit Ticket Sample Solutions

1. Solve $\sqrt{2x+15} = x + 6$. Verify the solution(s).

$$2x + 15 = x^2 + 12x + 36$$
$$0 = x^2 + 10x + 21$$
$$0 = (x + 3)(x + 7)$$

The solutions are -3 and -7.

Check $x = -3$:

$$\sqrt{2(-3) + 15} = \sqrt{9} = 3$$
$$-3 + 6 = 3$$

So, -3 is a valid solution.

Check $x = -7$:

$$\sqrt{2(-7) + 15} = \sqrt{1} = 1$$
$$-7 + 6 = -1$$

Since $-1 \neq 1$, we see that -1 is an extraneous solution.

Therefore, the only solution to the original equation is -3.

2. Explain why it is necessary to check the solutions to a radical equation.

Raising both sides of an equation to a power can produce an equation whose solution set is not equivalent to that of the original equation. In the problem above, $x = -7$ does not satisfy the equation.

Problem Set Sample Solutions

Solve.

1. $\sqrt{2x-5} - \sqrt{x+6} = 0$

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2. $\sqrt{2x-5} + \sqrt{x+6} = 0$

   No solution

3. $\sqrt{x-5} - \sqrt{x+6} = 2$

   No solution

4. $\sqrt{2x-5} - \sqrt{x+6} = 2$

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5. $\sqrt{x+4} = 3 - \sqrt{x}$

   $\frac{25}{36}$

6. $\sqrt{x+4} = 3 + \sqrt{x}$

   No solution

7. $\sqrt{x+3} = \sqrt{5x+6} - 3$

   6

8. $\sqrt{2x+1} = x - 1$

   4

9. $\sqrt{x+12} + \sqrt{x} = 6$

   4

10. $2\sqrt{x} = 1 - \sqrt{4x-1}$

   $\frac{1}{4}$

11. $2x = \sqrt{4x - 1}$

$\frac{1}{2}$

12. $\sqrt{4x - 1} = 2 - 2x$

$\frac{1}{2}$

13. $x + 2 = 4\sqrt{x - 2}$

6

14. $\sqrt{2x - 8} + \sqrt{3x - 12} = 0$

4

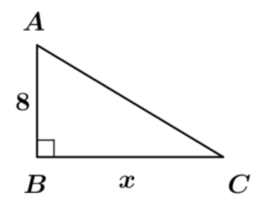
15. $x = 2\sqrt{x - 4} + 4$

4, 8

16. $x - 2 = \sqrt{9x - 36}$

5, 8

17. Consider the right triangle ABC shown to the right, with $AB = 8$ and $BC = x$.



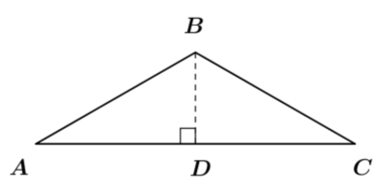
a. Write an expression for the length of the hypotenuse in terms of x.

$AC = \sqrt{64 + x^2}$

b. Find the value of x for which $AC - AB = 9$.

The solutions to the mathematical equation $\sqrt{64 + x^2} - 8 = 9$ *are* -15 *and* 15. *Since lengths must be positive,* -15 *is an extraneous solution, and* $x = 15$.

18. Consider the triangle ABC shown to the right where $AD = DC$, and $\overline{BD}$ is the altitude of the triangle.



a. If the length of $\overline{BD}$ is x cm, and the length of $\overline{AC}$ is 18 cm, write an expression for the lengths of $\overline{AB}$ and $\overline{BC}$ in terms of x.

$AB = BC = \sqrt{81 + x^2}$ cm

b. Write an expression for the perimeter of $\triangle ABC$ in terms of x.

$\left(2\sqrt{81 + x^2} + 18\right)$ cm

c. Find the value of x for which the perimeter of $\triangle ABC$ is equal to 38 cm.

$\sqrt{19}$ cm