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GRADE 2 • MODULE 7

Problem Solving with Length, Money, and Data

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Grade 2 • Module 7

Problem Solving with Length, Money, and Data

OVERVIEW

Module 7 presents an opportunity for students to practice addition and subtraction strategies within 100 and problem-solving skills as they learn to work with various types of units within the contexts of length, money, and data. Students represent categorical and measurement data using picture graphs, bar graphs, and line plots. They revisit measuring and estimating length from Module 2 but now use both metric and customary units.

Module 7 opens with students representing and interpreting categorical data. In Grade 1, students learned to organize and represent data with up to three categories. Now, in Grade 2, students build upon this understanding by drawing both picture and bar graphs (**2.MD.10**). First, they record category counts in a table, solving problems based on the information in the table. Next, they draw picture graphs in which each picture represents one object. Finally, they represent the same data set in the form of a bar graph, where one axis names the categories and the other shows a single-unit count scale. Students use the information to solve *put together*, *take apart*, and *compare* problems (**2.MD.10**), making connections to finding sums and differences on a number line diagram. In the final lesson of Topic A, students display money data in the form of a bar graph, thus establishing a connection to word problems with coins in Topic B.

In Topic B, students work with the most popular units of all: bills and coins. Students apply their knowledge of coin values, place value strategies, and the properties of operations to solve addition and subtraction word problems (**2.NBT.5**, **2.MD.8**) to find the total value of a group of coins or bills. Next, they use coins to find multiple ways to represent the same quantity, sometimes using the fewest number of coins. Students then focus on the decomposition of a dollar, where they see that this unit behaves like all others they have seen before (e.g., 100 ones = 1 hundred, 100 cm = 1 m). Students learn how to make change from one dollar using counting on, simplifying strategies (e.g., number bonds), and the relationship between addition and subtraction. As students use coins or bills to solve addition and subtraction word problems within 100,¹ they use drawings and equations to represent the unknown in various situations. The Application Problems throughout this module include solving two-step word problems involving two-digit money amounts (e.g., \$28 + \$47 or 28¢ + 47¢), as students use this new context to increase fluency with addition and subtraction within 100 (**2.NBT.5**).

After the Mid-Module Assessment, Topic C reviews the measurement concepts and skills presented in Module 2, now with a focus on customary units. Students deepen their understanding of a *length unit* as they lay one-inch square tiles end-to-end to create simple inch rulers, just as they created centimeter rulers in Module 2. They see again that the smaller the unit, the more iterations are necessary to cover a given distance. Students measure the length of various objects with their new unit rulers (**2.MD.1**), applying

¹Totals are limited to within 100 cents, or 1 dollar, when working with coins, and 100 dollars when working with bills.

important concepts such as the understanding that the zero point on a ruler is the beginning of the total length and the number on a ruler means the distance covered by that number of length units.

In Topic D, students apply their measurement skills and knowledge of the ruler to measure a variety of objects using the appropriate measurement tools, such as inch rulers and yardsticks, just as they measured with centimeter rulers, meter sticks, and meter tapes in Module 2 (**2.MD.1**). Students thereby add to their bank of benchmark lengths, such as an inch being the distance across a quarter. By doing so, students develop mental images of an inch, a foot, or a yard, which empowers them to estimate a given length (**2.MD.3**).

In addition, in Topic D, students measure objects using both metric and customary length units, thereby developing an understanding of how the number of units needed depends upon the size of the unit chosen (**2.MD.2**). As in Topic C, students recognize, for example, that the smaller the length unit, the more iterations are necessary to cover a given distance. Topic D concludes with students measuring to determine how much longer one object is than another (**2.MD.4**). Students use addition and subtraction to compare two lengths, subtracting the length of the shorter object from the length of the longer object to determine the difference (e.g., $40 \text{ in} - 35 \text{ in} = 5 \text{ in}$, or $35 \text{ in} + \underline{\quad} = 40 \text{ in}$).

Whereas in Topic D students used rulers to compare lengths, in Topic E, students use drawings (e.g., tape diagrams and number bonds) and equations with an unknown to represent addition and subtraction word problems (**2.MD.5**). Once they have a solid conceptual understanding of length, students are ready to represent whole numbers as lengths on a number line (**2.MD.6**) and apply their knowledge of the ruler to a number line diagram. In Topic E, they are asked to identify unknown numbers on a number line by using place value, reference points (e.g., 5, 10, 25, and 50), and the distance between points. Students are also asked to represent two-digit sums and differences using the number line as a measurement model for combining and comparing lengths.

Topic F follows naturally, with students generating measurement data and representing it with a line plot (**2.MD.9**). Students position data along a horizontal scale with whole number markings, drawn as a number line diagram (**2.MD.6**). Since students are working with length, the scale on their line plots corresponds to the scale on their rulers. After generating measurement data, students create line plots from different data sets, and then they discuss and interpret the results.

The Mid-Module Assessment follows Topic B, and the End-of-Module Assessment follows Topic F.

Notes on Pacing for Differentiation

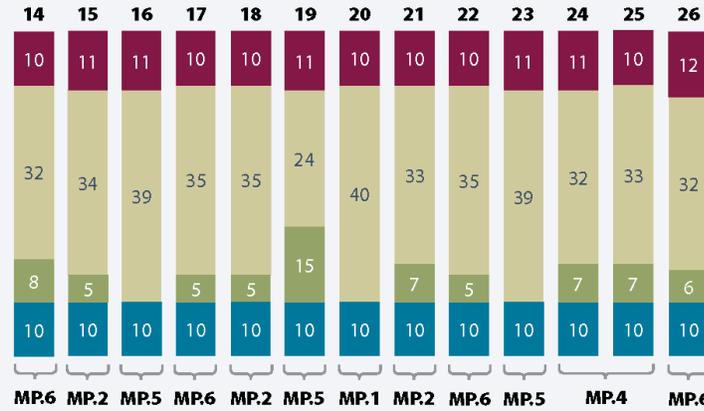
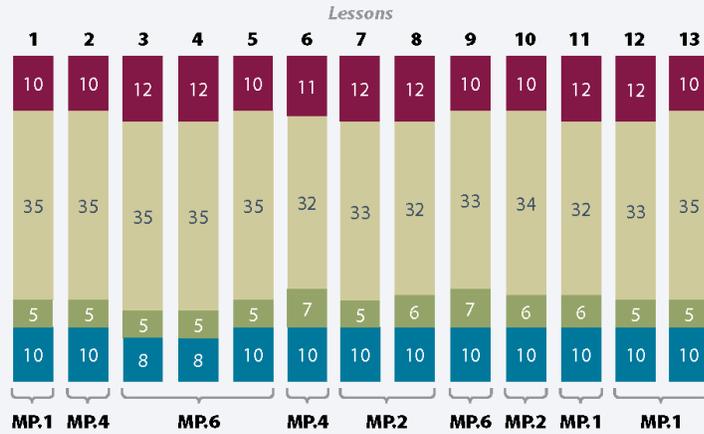
If pacing is a challenge, consider consolidating Lessons 1 and 2, Lessons 3 and 4, Lessons 11 and 12, and Lessons 14 and 15. Omit Lesson 26, or consolidate it with Lesson 25.



Distribution of Instructional Minutes

This diagram represents a suggested distribution of instructional minutes based on the emphasis of particular lesson components in different lessons throughout the module.

- Fluency Practice
- Concept Development
- Application Problems
- Student Debrief



MP = Mathematical Practice

Focus Grade Level Standards

Use place value understanding and properties of operations to add and subtract.

- 2.NBT.5** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Measure and estimate lengths in standard units.

- 2.MD.1** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
- 2.MD.2** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

- 2.MD.3** Estimate lengths using inches, feet, centimeters, and meters.
- 2.MD.4** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

- 2.MD.5** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
- 2.MD.6** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.²

- 2.MD.8** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

- 2.MD.9** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
- 2.MD.10** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Foundational Standards

- 1.MD.2** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*
- 1.MD.4** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
- 2.OA.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See CCSS-M Glossary, Table 1.)

²Focus on money. Time is taught in Module 8.

- 2.NBT.2** Count within 1000; skip-count by $5s^3$, 10s, and 100s.
- 2.NBT.4** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.
- 2.NBT.6** Add up to four two-digit numbers using strategies based on place value and properties of operations.

Focus Standards for Mathematical Practice

- MP.1** **Make sense of problems and persevere in solving them.** Students draw to determine the part-whole relationships embedded within various word problem types, and based on their analysis, they persevere to use various addition and subtraction strategies to solve problems. They then persist in making a statement of the solution to answer the question in the original context. In this module, the problem-solving contexts involve length, money, and data.
- MP.2** **Reason abstractly and quantitatively.** Students compare measurements using rulers, tape diagrams, and graphs. After they abstract the number of units or length measurements to calculate differences, they reinterpret the difference using the given units within a problem. Students also abstract the value from a set of coins to find the total value and then express that value once again in terms of dollars or cents.
- MP.4** **Model with mathematics.** Students create drawings (e.g., tape diagrams) and write equations to model and solve word problems involving units of length, money, and data. Students use appropriate representations (e.g., line plot, bar graph, and picture graph) to visually display data. Students also use the number line to understand numbers and their relationships and to represent sums and differences within 100. Students organize their thinking about money by modeling with dollars and coins to solve addition and subtraction word problems.
- MP.5** **Use appropriate tools strategically.** Students apply their measurement skills and knowledge of the ruler to measure a variety of objects using the appropriate measurement tools, such as inch rulers and yardsticks. When conventional measurement tools are not available, students make decisions about which resources might be helpful, such as using iteration with a shoe, a book, or a lima bean, while recognizing the limitations of such tools.
- MP.6** **Attend to precision.** Students attend to precision when they iterate a physical unit to create inch rulers. They align the zero point on a ruler as the beginning of the total length, and they use various measurement tools and precise language to describe their experience: “I used an inch as the length unit.” Students learn estimation strategies for measurement and make closer and closer approximations to the actual length. They assign specific values to different coins and count up, starting with the largest value. Students generate and represent data in a bar graph, picture graph, or line plot, labeling axes appropriately and specifying the unit of measure.

³Use an analog clock to provide a context for skip-counting by fives.

Overview of Module Topics and Lesson Objectives

| Standards | Topics and Objectives | Days |
|---|--|------|
| 2.MD.10 2.MD.6 | A Problem Solving with Categorical Data Lesson 1: Sort and record data into a table using up to four categories; use category counts to solve word problems. Lesson 2: Draw and label a picture graph to represent data with up to four categories. Lesson 3: Draw and label a bar graph to represent data; relate the count scale to the number line. Lesson 4: Draw a bar graph to represent a given data set. Lesson 5: Solve word problems using data presented in a bar graph. | 5 |
| 2.NBT.5 2.MD.8 2.NBT.2 2.NBT.6 | B Problem Solving with Coins and Bills Lesson 6: Recognize the value of coins and count up to find their total value. Lesson 7: Solve word problems involving the total value of a group of coins. Lesson 8: Solve word problems involving the total value of a group of bills. Lesson 9: Solve word problems involving different combinations of coins with the same total value. Lesson 10: Use the fewest number of coins to make a given value. Lesson 11: Use different strategies to make \$1 or make change from \$1. Lesson 12: Solve word problems involving different ways to make change from \$1. Lesson 13: Solve two-step word problems involving dollars or cents with totals within \$100 or \$1. | 8 |
| | Mid-Module Assessment: Topics A–B (assessment ½ day, return ½ day, remediation or further applications 1 day) | 2 |
| 2.MD.1 | C Creating an Inch Ruler Lesson 14: Connect measurement with physical units by using iteration with an inch tile to measure. Lesson 15: Apply concepts to create inch rulers; measure lengths using inch rulers. | 2 |

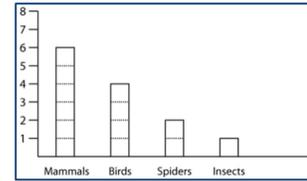


| Standards | Topics and Objectives | Days |
|--|---|-----------|
| <p>2.MD.1 2.MD.2 2.MD.3 2.MD.4</p> | <p>D Measuring and Estimating Length Using Customary and Metric Units</p> <p>Lesson 16: Measure various objects using inch rulers and yardsticks.</p> <p>Lesson 17: Develop estimation strategies by applying prior knowledge of length and using mental benchmarks.</p> <p>Lesson 18: Measure an object twice using different length units and compare; relate measurement to unit size.</p> <p>Lesson 19: Measure to compare the differences in lengths using inches, feet, and yards.</p> | 4 |
| <p>2.MD.5 2.MD.6 2.NBT.2 2.NBT.4 2.NBT.5</p> | <p>E Problem Solving with Customary and Metric Units</p> <p>Lesson 20: Solve two-digit addition and subtraction word problems involving length by using tape diagrams and writing equations to represent the problem.</p> <p>Lesson 21: Identify unknown numbers on a number line diagram by using the distance between numbers and reference points.</p> <p>Lesson 22: Represent two-digit sums and differences involving length by using the ruler as a number line.</p> | 3 |
| <p>2.MD.6 2.MD.9 2.MD.1 2.MD.5</p> | <p>F Displaying Measurement Data</p> <p>Lesson 23: Collect and record measurement data in a table; answer questions and summarize the data set.</p> <p>Lesson 24: Draw a line plot to represent the measurement data; relate the measurement scale to the number line.</p> <p>Lessons 25–26: Draw a line plot to represent a given data set; answer questions and draw conclusions based on measurement data.</p> | 4 |
| | <p>End-of-Module Assessment: Topics A–F (assessment ½ day, return ½ day, remediation or further applications 1 day)</p> | 2 |
| Total Number of Instructional Days | | 30 |

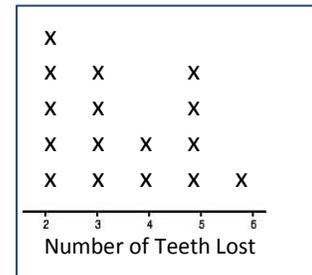
Terminology

New or Recently Introduced Terms

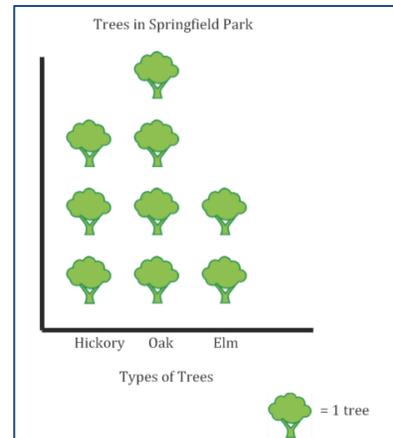
- Bar graph (pictured to the right)
- Category (a group of people or things sharing a common characteristic; e.g., bananas are in the fruit category)
- Data (a set of facts or pieces of information)
- Degree (unit used to measure temperature, e.g., degrees Fahrenheit)
- Foot (ft, a unit of length equal to 12 inches)
- Inch (in, a unit of length)
- Legend (the notation on a graph explaining what symbols represent)
- Line plot (a graphical representation of data—pictured to the right)
- Picture graph (a representation of data like a bar graph, using pictures instead of bars—pictured to the right)
- Scale (a number line used to indicate the various quantities represented in a bar graph—pictured below to the right)
- Survey (collecting data by asking a question and recording responses)
- Symbol (a picture that represents something else)
- Table (a representation of data using rows and columns)
- Thermometer (a tool used to measure temperature)
- Yard (yd, a unit of length equal to 36 inches or 3 feet)



Bar Graph



Line Plot



Picture Graph

Familiar Terms and Symbols⁴

- Benchmark number (e.g., numbers like the multiples of 10)
- Centimeter (cm, a unit of length measure)
- Cents (e.g., 5¢)
- Coins (e.g., penny, nickel, dime, and quarter)
- Compare
- Compose
- Decompose
- Difference
- Dollars (e.g., \$2)
- Endpoint



Scale

⁴These are terms and symbols students have seen previously.

- Equation
- Estimation (an approximation of the value of a quantity or number)
- Hash mark (the marks on a ruler or other measurement tool)
- Height
- Length
- Length unit
- Meter (m, a unit of length measure)
- Meter strip, meter stick
- Number bond
- Number line (a line marked at evenly spaced intervals)
- Overlap (to extend over or cover partly)
- Ruler
- Tally mark
- Tape diagram
- Unit
- Value

Suggested Tools and Representations

- Bar graph (representation of data)
- Centimeter cube
- Centimeter ruler
- Dice
- Grid paper
- Inch and centimeter ruler
- Inch tiles
- Line plot
- Measuring tape
- Meter stick
- Money (i.e., dollars, coins)
- Number bond
- Number line
- Personal white board
- Picture graph
- Table
- Tape diagram
- Yardstick

Scaffolds⁵

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units*, please refer to “How to Implement *A Story of Units*.”

Assessment Summary

| Type | Administered | Format | Standards Addressed |
|-------------------------------|---------------|----------------------------------|--|
| Mid-Module Assessment Task | After Topic B | Constructed response with rubric | 2.NBT.5 2.MD.8 2.MD.10 |
| End-of-Module Assessment Task | After Topic F | Constructed response with rubric | 2.NBT.5 2.MD.1 2.MD.2 2.MD.3 2.MD.4 2.MD.5 2.MD.6 2.MD.8 2.MD.9 2.MD.10 |

⁵Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website www.p12.nysed.gov/specialed/aim for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.